A Price-Forecasting Model

In this section, a price-forecasting model is developed by applying an inverse demand system approach. Assuming that there are *n* goods in a demand system, let *q* denote an *n*-coordinate column vector of quantities demanded for a "representative" consumer, *p* an *n*-coordinate vector of the corresponding prices for the *n* goods, $m = p'q$ the consumer's expenditure, and $U(q)$ the utility function, which is assumed nondecreasing and quasi-concave in *q*. The primal function for maximizing consumer utility is the Lagrangian function:

$$Maximize \;\; L = U(q) - \lambda (p'q - m). \qquad (1)$$

By differentiating the Lagrangian function, the necessary conditions for optimums are

$$u_i (q) = \lambda \, p_i \qquad i = 1, 2,..., n \qquad (2)$$

$$p' q = m \qquad (3)$$

in which $u_i (q)$ is the marginal utility of the *i*th good. In equation 2, λ is known as the marginal utility of income, showing the change in the maximized value of utility as income changes. This equation represents an equilibrium condition, in which each marginal utility divided by its price is equal (constant at λ) for all goods.

The inverse demand system can be obtained by eliminating the Lagrangian multiplier λ from the necessary conditions of equation 2. Multiplying both sides of equation 2 by q_i and summing over *n* goods to satisfy the budget constraint of equation 3, the Lagrangian multiplier is then obtained as

$$\lambda = \Sigma_j \, q_j \, u_j \, (q) \, / \, m \qquad j = 1, 2, .., n \qquad (4)$$

Substituting equation 4 into equation 2 yields the Hotelling-Wold identity, which defines the inverse demand system from a differentiable direct utility function as

$$p_i \, / \, m = u_i \, (q) \, / \, \Sigma_j \, q_j \, u_j \, (q) \qquad i, j = 1, 2, .., n \quad (5)$$

in which p_i / m is the normalized price of the *i*th commodity.

Equation 5 represents an inverse demand system in which the variation of price is a function of quantities demanded and is proportional to a change in income. For given quantities demanded, an increase in income will cause each commodity's price to increase at the same rate. Therefore, all income flexibilities are implicitly constrained to 1. This model has been applied in Huang (1991) for a 40-equation demand system consisting of 39 food categories and 1 nonfood sector.

On the choice of functional form for equation 5, the loglinear approximation of the Hotelling-Wold identity is used in this study for practical reasons. In addition to the linear model for easy estimation, the parameters of the loglinear form represent direct estimates of demand flexibilities. An annual statistical model for the *i*th price equation in terms of *n* quantities demanded is specified as follows:

$$\log (p_{i\,t} \, / \, m_t) = \alpha_i + \Sigma_j \, \beta_{i\,j} \, \log (q_{j\,t}) + v_{i\,t} \qquad i , j = 1, 2, .., n \qquad (6)$$

where variables at time *t* are $p_{i\,t}$ (price of *i*th commodity), m_t (per capita income), $q_{j\,t}$ (quantity demanded for *j*th commodity); $v_{i\,t}$'s are random disturbances.

Furthermore, according to Houck (1966) and Huang (1994), the price flexibilities of $\beta_{i\,j}$'s should be constrained by the following theoretical relationships:

$$\beta_{i\,j} = (w_j \, / \, w_i) \, \beta_{j\,i} - w_j \, (\Sigma_k \, \beta_{j\,k} - \Sigma_k \, \beta_{i\,k}) \qquad i, j, k = 1, 2, .., n \qquad (7)$$

where w_i is the expenditure share of the *i*th food category.

As suggested by Muth (1961), there is little empirical interest in assuming that the disturbance term in a structural model is completely unpredictable, and it is desirable to assume that part of the disturbance may be predicted from past observations. Because the expected values of the disturbance could be related to economic conditions prevailing in the past years, the disturbance is assumed to be not independent over time but to follow an autoregressive process.

Following Muth's suggestion, an autoregressive specification for the disturbance terms of the inverse demand system in equation 6 is applied in this study to enhance the price-forecasting capability. An autoregressive process of residuals lagged up to *l* years is specified as follows:

$$v_{i\,t} = \Sigma_h \gamma_{i\,h} v_{i\;t\text{-}h} + \varepsilon_{i\,t}$$
$$i = 1, 2, .., n;\;\; h = 1, 2, .., l \qquad (8)$$

where $\varepsilon_{i\,t}$'s are random disturbances in which $\varepsilon_{i\,t}$ is assumed to be identical normal and independently distributed as $\varepsilon_{i\,t} \sim IN(0, \delta^2 I)$, and $v_{i\,t}$ is assumed to be serially correlated.

In the following empirical application, a structural component model of equation 6 is estimated. In addition, for an improvement in forecasting performance, an autoregressive model is estimated by incorporating the disturbance specification of equation 8.