EDMP Methodology

Review of Literature

There have been a number of contributors to the development of equilibrium displacement modeling. The literature is usually traced back to Muth (1964) who developed the reduced forms for proportional displacements from equilibrium for a system of equations of supply and demand for a product dependent on two factors of production and exogenous shifters for each of the functions. However, in a 1958 article in the *Journal of Farm Economics*, Buse (1958) demonstrated the development of what he called "total elasticities"—the reduced-form elasticities of a system of supply and demand equations for two commodities similar to that later devised by Muth—and contrasted his "total elasticities" with Marshallian *ceteris paribus* elasticities. Buse's was the first article to use matrix algebra to state and solve his system of equations. Gardner (1975) employed a formulation identical to Muth's to investigate the relationship of retail food prices to farm prices. Sumner and Wohlgenant (1985) first applied the term "equilibrium displacement modeling" to Muth's formulation. Wohlgenant (1993) also extended Muth's formulation to multistage industries. Piggott et al. (1995) employed the term "equilibrium displacement modeling" and formulated their model in matrix algebra. Davis and Espinoza (1998) extended the Gardner analysis to develop the full distribution of parameter values rather than only selected values. Most recently, Sumner (2005) used equilibrium displacement methods to assess the effects of U.S. commodity polices on world prices and trade.

Samuelson (1952) first demonstrated that the spatial equilibrium problem could be cast and solved as a linear programming (LP) problem. Takayama and Judge (1971) demonstrated how quadratic programming could be used to solve linear supply and demand equations, determining both prices and quantities endogenously. However, Plessner (1965) and Yaron et al. (1965) had earlier applied quadratic programming methods to price-endogenous modeling of the U.S. agricultural sector. Because of the scarcity and cost of quadratic programming solution algorithms, early MP literature of price-endogenous models in agricultural economics turned to LP methods. Martin (1972) incorporated stepped supply and demand functions in LP models. Martin's method significantly increased the dimensionality of LP problems because it required a row and column for each step of the supply and demand schedules. However, Miller (1963) had earlier published a method of incorporating sloping demand and supply functions in LP models by selecting among activities representing the area (price times quantity) under each step of the functions. Miller's method required a column for each step but required only a single convex combination constraint, thus reducing the model dimensionality significantly from that required by Martin's formulation.

With the advent of efficient and affordable quadratic programming algorithms in the early 1970s, price-endogenous MP modeling rapidly adopted quadratic programming methods. Harrington (1973) combined price-endogenous quadratic programming modeling and input/output analysis to develop a forerunner of today's computable general equilibrium models.

The literature of positive MP is replete with applications but only two methodological articles. Howitt (1995) explains a pragmatic method of using dual values of LP model solutions to introduce quadratic terms that assure that the model's base period solution matches the base period primal variable levels of the system. An additional advantage of Howitt's positive MP is that it eliminates most corner solutions; hence the model adjusts gradually and proportionally to changes in prices, rather than abruptly shifting from one corner solution to another. Preckel et al. (2002) extend the Howitt positive MP methods to calibrate both the primal and dual levels of the system. They apply their method to calibrating base period prices and quantities in a system of agricultural sector supply and demand relationships.

The asset-fixity or investment-disinvestment literature—most closely associated with Johnson and Hardin (1955), Johnson and Quance (1972), and Schmid (1997)—is central to specifying the supply response of the model. The asset-fixity paradigm is predicated upon there being a gap between the cost of investing in an additional unit of durable capital (its acquisition cost) and the return from disinvesting in it (its salvage value). When the marginal value product of a capital item is within this range, it is considered fixed but allocatable, while outside the range, it is considered variable. Not all capital is either fixed or variable in a problem; but, different items can be at their acquisition costs, at their salvage values, or in their fixed but allocatable range in between. Under the asset-fixity hypothesis, the length of run is endogenized separately for each different type of capital. In the 1980s, there was some controversy over whether the asset-fixity theory was a defensible viewpoint; but Chavas (1994) rigorously demonstrated Johnson's underlying premises under sunk costs and temporal uncertainty.

Theoretical Development

In specifying the theoretical model, we first start with the equilibrium displacement method, recast it in the positive MP framework, discuss the modeling of the supply side with the asset-fixity paradigm, then complete the EDMP model with consistent aggregation/disaggregation of the demand side. We note certain limitations of each of these building blocks that may be strongly determinative of model performance.

The equilibrium displacement methodology starts with a standard set of economic structural equations of supply and demand.

Structural equations:

$D_c = e_{c,pc} P_c$	$+ e_{c,pL} P_L$	$+ e_w W$	Crop demands	1 . . n	
$Sc = \varepsilon_{c,pc} P_c$	$+ \varepsilon_{c,pL} P_L$	$+ \varepsilon_x X$	Crop supplies	1 . . n	
$D_L = e_{L,pc} P_c$	$+ e_{L,pL} P_L$	$+ e_y Y$	Livestock demands	1 . . m	(1)
$S_L = \varepsilon_{L,pc} P_c$	$+ \varepsilon_{L,pL} P_L$	$+ \varepsilon_z Z$	Livestock supplies	1 . . m	
$Q_c = Dc = Sc$			Crops market clearing	1 . . n	
$Q_L = D_L = S_L$			Livestock market clearing	1 . . m	

Where:

D_c = Crop demands, D_L = Livestock demands

S_c = Crop supplies, S_L = Livestock supplies

P_c = Prices of crop commodities, 1 . . n

P_L = Prices of livestock commodities, 1 . . m

Q_c = Quantities of crop commodities, 1 . . n

Q_L = Quantities of livestock commodities, 1 . . m

W = Exogenous factors influencing crop demands

X = Exogenous factors influencing crop supplies

Y = Exogenous factors influencing livestock demands

Z = Exogenous factors influencing livestock supplies

e.,. = elasticities of demand w.r.t subscripted variables

ε.,. = elasticities of supply w.r.t subscripted variables

Substitute displacements from equilibrium: D*, S*, P*, Q*, W*, X*, Y*, and Z* for respective variables. For example, $D^* = (D_{scenario} - D_{equilibrium})$.

Substitute market clearing equations into S and D equations.

Rearrange so that Q*s and P*s are functions of exogenous variables: W*, X*, Y*, and Z*.

$$
\begin{aligned}
Q^*_c &= e_{c,pc} P^*_c + e_{c,pL} P^*_L + e_w W^* \\
Q^*_c &= \varepsilon_{c,pc} P^*_c + \varepsilon_{c,pL} P^*_L + \varepsilon_x X^* \qquad (2) \\
Q^*_L &= e_{L,pc} P^*_c + e_{L,pL} P^*_L + e_y Y^* \\
Q^*_L &= \varepsilon_{L,pc} P^*_c + \varepsilon_{,pL} P^*_L + \varepsilon_z Z^*
\end{aligned}
$$

Arrange above equations in matrix form:

$$
\begin{array}{cccccccccccc}
I_n & 0_m & -e_{c,pc} & -e_{c,pL} & Q^*_c & & e_w & 0 & 0 & 0 & W^* & \\
I_n & 0_m & -\varepsilon_{L,pc} & -\varepsilon_{L,pL} & Q^*_L & = & 0 & \varepsilon_x & 0 & 0 & X^* & (3) \\
0_n & I_m & -e_{c,p} & -e_{c,p} & P^*_c & & 0 & 0 & e_y & 0 & Y^* & \\
0_n & I_m & -e_{L,pc} & -e_{L,pL} & P^*_L & & 0 & 0 & 0 & e_z & Z^* & \\
 & \Gamma & * & & \Delta & = & & B & * & & \Omega &
\end{array}
$$

Then solve for Δ: $\Delta = \Gamma^{-1} B\, \Omega = \Pi\, \Omega$

$\Pi = \Gamma^{-1} B$ are generally termed reduced-form elasticities of endogenous response.

Assumptions of Equilibrium Displacement Models

As noted by Piggott et al. (1995), equilibrium displacement models rest on four key assumptions:

1. Elasticities of endogenous supply and demand relationships are known and constant.
2. Elasticities of supplies and demands, with respect to exogenous variables, are known and constant.
3. Technology of production is known and constant.
4. Displacements are restricted to be in the neighborhood of equilibrium.

Limitations of Equilibrium Displacement Models

Those assumptions are also the Achilles heel of equilibrium displacement models:

1. Adjustment scenarios often entail changing any or all of the above assumptions, analyzing large displacement from the initial equilibrium, and/or determining base equilibrium values from indirect data, thus complicating their application.
2. Expansionary displacements assume that no physical constraints to expansion exist, for example, no limitations on total cropland or limitations on existing production capacity.
3. Some contractionary displacements can exceed 100 percent of the base level of the activity. Such solutions are *a priori* infeasible because they imply the process in question is operating in reverse.
4. Equilibrium displacement model supply functions are assumed to be downwardly continuous, whereas a correct specification requires that each supply function be truncated at the point where its supply price drops below its average variable cost.
5. Neither expansionary nor contractionary displacements can be guaranteed to be on the efficient frontier of the underlying production/demand functions, but may be either interior points or infeasible points.
6. In the constant elasticity equilibrium displacement formulation, it is not conceptually possible to calculate a correct monopolistic/monopsonistic maximum quasi-rent solution. Quasi-rents change monotonically upward or downward, with successive restrictions in output, depending on whether demand is inelastic or elastic.

To overcome these limitations, we adopt an MP implementation of the equilibrium displacement model.

The EDMP Formulation

We redefine the constant elasticity equilibrium displacement problem to one of comparing successive equilibria of a system of linear (constant slope) supply and demand functions with quadratic programming.

Following Preckel et al. (2002):

Max: $Z = F'x - 1/2\, x'\, H\, x.$ (4)

Z is the objective function to be maximized. Z can be either the sum of consumer plus producer surpluses or the sum of residual quasi-rents, depending on whether the model is perfectly competitive or monopolistic.

Subject to:

$A_{11}x$	=	Free	Indicator accounts,	(4a)
$A_{21}x$	$\leq$	b	Technical constraints,	(5)
$I_{31}x$	=	c	Calibration constraints, and	(6)
x	$\geq$	0	Non-negativity constraint.	(7)

Where:

A_{11}, A_{21}	=	A matrix of Leontief technical requirements of processes
I_{31}	=	An identity matrix of calibration constraints, suspended after calibration
x	=	A vector of optimized variables (which assure that all solutions are feasible and efficient)
b	=	A vector of right hand sides of constraints
c	=	A vector of calibration targets to reproduce base equilibrium, suspended after calibration
F	=	A vector of intercepts of supply and demand processes
H	=	Hessian matrix of marginal adjustment costs and demand slopes, assumed to be positive semidefinite for maximization.

Equation 4a is necessary because the value of the objective function, equation 4, is confounded by perturbations necessary to calibrate the model to the base period prices and quantities. Similarly, to model some agricultural policies, it may be necessary to define processes differently from observed supply and demand relationships. Any such changes need to be backed out of the model solutions to reflect true supply and demand prices and quantities.

Equation 6, which contains the quantity targets of the equilibrium solution, is enforced only in the initial calibration solutions. When the model is calibrated to the desired accuracy, its optimal solution will return exactly the quantities specified in equation 6, without any quantity constraints. After that, equation 6 is suspended to allow the model to adjust all prices and quantities simultaneously in response to changes in the scenario. Thus, differences of scenario solutions from the base solution enforced in equation 6 are equilibrium displacements under the assumption of constant slope relationships rather than constant elasticity relationships. The constant slope formulation inherent in EDMP models has the advantage of being theoretically consistent with monopolistic maximization of quasi-rents, in contrast to constant elasticity equilibrium displacement models.

Equivalence of EDMP Formulation to a Profit Function Formulation

Monopolistic Firm and Competitive Industry Cases

Let F(x) be a general multi-output multi-input profit function, with x containing both inputs (-) and outputs (+) and with prices related to quantities, subject to equations 5 and 7.

A second order Taylor series expansion of F(x) in the neighborhood of its maximum (x*) is:

$$F(x) = F(x^*) + \frac{F'(x^*)(x-x^*)}{1!} + \frac{F''(x^*)(x-x^*)^2}{2!} + R, \qquad (8)$$

where F' and F'' are the first and second derivatives of F(x*) and R represents the higher order nonlinearities of F(x).

By definition, F''(x*) is the Hessian, H(x*).

Assuming the base situation to be in equilibrium (a maximum), then F'(x*) = 0. Rearranging terms to matrix notation, the Taylor series expansion becomes:

$$F(x) = F(x^*) + 1/2(x-x^*)' \, H \, (x-x^*), \qquad (9)$$

where the Hessian matrix is assumed to be negative semidefinite. Changing the sign in equation 9 allows the Hessian to be specified as positive semidefinite. From equation 9, it is clear that, in the monopolistic case, the equilibrium displacement maximand, Z, once calibrated, is identical to the monopolistic firm's profit function.

The model can be solved either monopolistically (that is, for a firm with market power) by equating marginal factor costs to marginal revenues or perfectly competitively by maximizing the sum of producers' plus consumers' surpluses (that is, for a perfectly competitive firm or an industry). Both monopolistic and perfectly competitive behavior can be combined for different activities within a single model. (For applications of EDMP to mixed competitive and monopolistic problems, see Jefferson-Moore and Harrington (2006) and Harrington and Jefferson-Moore (2007).)

Figure 1 illustrates the EDMP perfectly competitive supply and demand equilibrium for a commodity. The gradient is the perfectly competitive market price, and residual rents are identically equal to zero. Figure 2 illustrates monopolistic/monopsonistic equilibrium, found by equating marginal revenue with marginal factor cost. Factor and product prices are found on the original factor supply and product demand functions. Residual rents, shown as the shaded area, are at a maximum.

Figure 1

EDMP purely competitive supply and demand of commodity

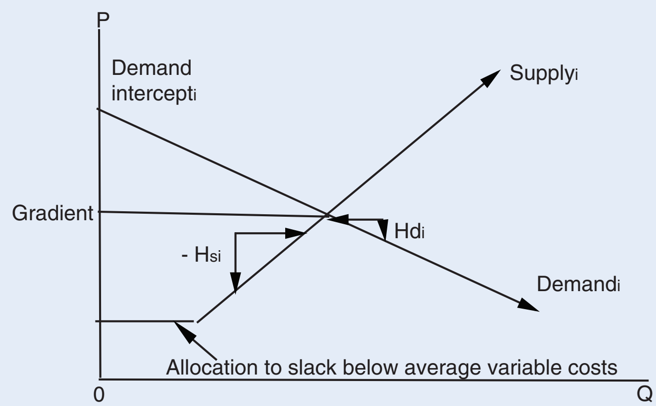

Figure 2

Monopolistic/monopsonistic supply and demand