Model Equation System

The following section presents a review of the modeling equation system, with model variables defined by equation. Model equations include: (1) an objection function that minimizes sector costs relating to manure hauling and land application, (2) balance equations that track stocks and flows of manure and manure nutrients, (3) constraints on land availability, distribution of confined animal farms (manure sources), and manure-nutrient application, and (4) cost accounting equations. Appendix 3 provides a listing of equation variables used below, with the names of corresponding GAMS model variables (Appendix 2) and model report variables (Appendix 4).

The regional optimization model minimizes the net cost of applying manure in the Chesapeake Bay watershed, subject to total manure produced, crop and pasture land available for manure spreading (onfarm and off-farm), and share of manure diverted to non-land-based uses. Net costs are defined as costs associated with manure land application, plus a penalty cost for manure that cannot be land-applied within the basin, less savings on reduced commercial fertilizer. The model allocates manure production in basin counties (*ct*) to spreadable land in destination counties (*ct2*), both within and outside the source county, to minimize the objective function expression (OBJ):

$$(1)\quad \mathrm{OBJ} = \sum_{ct}\sum_{ct2}[\mathrm{HAC}_{ct,ct2} + \mathrm{INC}_{ct2} + \mathrm{NM1}_{ct} + \mathrm{NM2}_{ct2} + \mathrm{ELA}_{ct} - \mathrm{FS}_{ct2}].$$

Costs include manure-hauling and application costs (HAC), manure-incorporation costs (INC), and nutrient management plan charges for source (NM1) and destination (NM2) counties (see equations 17 and 18 for cost items included). A penalty cost applied to manure levels' exceeding land application capacity in the basin (ELA) ensures that all manure is land-applied subject to available land (see equation 8 for calculation of surplus manure). However, the penalty cost is removed from the actual total cost value reported in the model solution report. Aggregate costs are further adjusted to reflect savings from reduced purchase and application costs for chemical fertilizers (FS). Net cost reported in the model solution is defined as total cost, net of savings from chemical fertilizer use.

In general, wet manure quantities are used to assess manure hauling and application costs, while manure-nutrient content and uptake determine the volume and direction of manure flows. Primary manure transfer equations are as follows:

$$(2)\quad \mathrm{M\_TRAN}_{ct,ct2} = ((\mathrm{M\_AP}_{ct,ct2,N*} * \mathrm{SH\_N}_{ct2}) + (\mathrm{M\_AP}_{ct,ct2,P*} * (1 - \mathrm{SH\_N}_{ct2}))) * \mathrm{AC\_SPR}_{ct,ct2}$$

$$(3)\quad \sum_{ct}\mathrm{AC\_SPR}_{\mathrm{ct,ct2}} \le Amax_{ct2} * \mathrm{WTAM}_{ct2}$$

$$(4)\quad \mathrm{M\_TRAN}_{\mathrm{ct,ct2}} = \sum_{gr}\sum_{sy}\sum_{ds}\mathrm{M\_TRN}_{ct,gr,ct2,sy,ds}$$

$$(5)\quad \sum_{ds}\mathrm{M\_TRN}_{ct,gr,ct2,sy,ds} \le \mathrm{M\_PRD}_{ct} * \mathrm{SH\_M}_{ct,gr,sy}$$

(where N^* represents a nitrogen standard and P^* represents a phosphorus standard, *gr* is county grid location, *sy* is manure system (lagoon, slurry, and dry), and *ds* is hauling distance interval in miles. Onfarm hauling distance is fixed based on average onfarm distance by county computed from the Census. Off-farm hauling distance is an endogenously derived, continuous variable falling within one of three distance intervals (0.5-2 miles, 2-10 miles, and more than 10 miles), with per-unit hauling cost dependent on the distance interval.

Manure transfers (M_TRAN) refer to manure hauled off the source farm and land-applied within or outside the county. In equation 2, dry manure tons by manure transfer is defined as the product of manure application rate (M_AP) by manure transfer—weighted by acreage shares under an N standard (SH_N) and P standard (1- SH_N)—and receiving acres (AC_SPR) in the destination county. Adjustments in applied manure per acre provide the link between restrictions on manure-nutrient use and manure transfers in the model.

Equation 3 restricts applied manure from all potential source counties to total spreadable acreage (Amax) in the destination county, adjusted for assumptions on land operator willingness to accept manure (WTAM). Equation 4 sets aggregate county-level manure transfers (M_TRAN) equal to the sum of manure transfers by source-county grid location (*gr*), system type (*sy*), and distance interval (*ds*). Equation 5 restricts source-county manure transfers by grid (*gr*) and system type (*sy*), based on the share (SH_M) of total manure production (M_PRD) across system type and grid, based on assignment procedures followed in the GIS.

Equations 6 through 8 balance manure production, onfarm surplus manure, manure transferred off-farm, and quantity of manure exceeding land application capacity in the basin.

$$(6)\quad \text{M\_SRP}_{ct} = \text{M\_PROD}_{ct} - \text{M\_ONFRM}_{ct}$$

$$(7)\quad \text{M\_USE}_{ct2} = \text{M\_ONFRM}_{ct2} + \sum_{ct} \text{M\_TRAN}_{ct,ct2}$$

$$(8)\quad \text{M\_ELA}_{ct} = \text{M\_SRP}_{ct} - \sum_{sy} \text{M\_IND}_{ct,sy} - \sum_{ct2} \text{M\_TRAN}_{ct,ct2}$$

Equation 6 sets county surplus manure to be moved off the farm (M_SRP) equal to manure production (M_PROD) less that used onfarm (M_ONFRM) in the source county. Equation 7 fixes manure use (M_USE) as onfarm manure use plus that quantity obtained from all off-farm sources (M_TRAN) in the destination county. Equation 8 sets the manure that exceeds land application capacity (M_ELA) within the assumed transport radius of a source county equal to the manure surplus in the source county less the sum of industrial uses (M_IND) and the sum of manure transfers out-of-county. Quantities of M_ELA manure are minimized in the model through the use of a penalty cost parameter that assigns a high cost to manure not land-applied in the basin.

Hauling distances for off-farm transfers are computed based on equations 9–11.

(9) $DS_{ct,gr,ct2} = [(\alpha_{ct,gr,ct2} * \delta^1_{ct,ct2}) + (\beta_{ct,ct2} * (AC\_ONF_{ct} + \sum_{ct} AC\_SPR_{ct,ct2}))] * \delta^2_{ct2}$

(10) $DS_{ct,gr,ct2} * M\_TRN_{ct,gr,ct2} = \sum_{sy} \sum_{ds} (DST_{ct,gr,ct2,sy,ds} * M\_TRN_{ct,gr,ct2,sy,ds})$

(11) $D\_MN_{ds} \leq DST_{ct,gr,ct2,sy,ds} \leq D\_MX_{ds}$

In equation 9, average hauling distance (DS) from source county (*ct*) and grid (*gr*) is calculated as a function of spreadable acres in the destination county (*ct2*). Off-farm hauling distance by manure transfer is computed based on acreage using manure from the source county (AC_SPR)—above a fixed acreage for onfarm manure use on confined animal farms (AC_ONF)—in the destination county. Intercept α and slope coefficient are β estimated from the GIS-derived linear regressions for within-county and out-of-county transfers.[18] The intercept term, representing linear hauling distance from the source farm for out-of-county transfers, is adjusted (δ^1) for selected county-to-county transfers to reflect significant natural barriers (e.g., large bodies of water). In addition, a circuity parameter (δ^2) is used to convert linear distance to road miles.[19] Thus, equation 9 establishes the key linkage in the model involving: (1) acreage accessed for manure spreading and (2) average hauling distance within and between counties, with values of each derived endogenously across county-transfer combinations.

[18] For in-county manure transfers, the intercept term of the area-to-distance relationship is set to zero.

[19] A fixed circuity parameter of 1.2 reflects an average of State-level parameters reported for the Chesapeake Bay watershed region (U.S. Department of Commerce, 1978).

In equation 10, average hauling distance (DS) from source-county grid to a given destination county represents a weighted average of hauling distances (DST) by manure-system type (*sy*) and distance interval (*ds*). This equation effectively integrates per-unit manure-hauling costs within area-to-distance relationships from the GIS, linking: (1) average hauling distance by county transfer with (2) individual hauls from source-county grid points. Minimum (D_MN) and maximum (D_MX) distance is specified by distance interval in equation 11, used in assessing per-unit costs.

Stocks and flows of manure nutrients (*np*)—nitrogen *n* or phosphorus *p*—are tied to manure quantities as follows:

(12) $M\_SRP_{ct} = NP\_EXC_{ct,np} / NP\_M_{ct,np}$

(13) $NP\_ONF_{ct2,np} = M\_ONFRM_{ct2} * NP\_M_{ct,np} \quad \text{where}_{ct} =_{ct2}$

(14) $NP\_TRN_{ct,ct2,np} = M\_TRAN_{ct,ct2} * NP\_M_{ct,np}$

Total excess manure nutrients (NP_EXC) are obtained from farm-level census data on manure production and onfarm assimilative capacity, aggregated to the county level. Equation 12 calculates manure surplus (M_SRP) based on pounds of excess N or P (*np*), depending on the nutrient standard in effect (*N** or *P**), and county-average nutrient content in lbs. per dry ton of manure (NP_M). In equation 13, onfarm manure nutrients (NP_ONF) reflect the quantity (M_ONFRM) and composition (NP_M) of manure produced and used on confined animal feeding operations. In equation 14, manure-nutrient flows (NP_TRN) are tied to manure transfers off the farm.

$$(15)\quad HAC_{ct,ct2} = \sum_{gr}\sum_{sy}\sum_{ds}[\,C1_{sy,ds} * (C2_{sy,ds} * DST_{ct,gr,ct2,sy,ds})\,]$$

$$* (M\_TRN_{ct,gr,ct2,sy,ds} / (1-(MS_{sy} + BED_{sy})\,]$$

$$(16)\quad INC_{ct2} = (\,C3 * SH\_I_{ct2} * (\,AC\_ONF_{ct2} + \sum_{ct} AC\_SPR_{ct,ct2}\,) * SH\_C_{ct2})$$

In equation 15, manure-hauling and application costs (HAC) are computed for onfarm and off-farm transfers. Costs reflect loading, unloading, and application costs per ton hauled (C1), hauling cost per ton-mile (C2), distance hauled (DST), and dry tons of manure hauled (M_TRN), adjusted for moisture content (MS) and bedding (BED). Hauling and application costs vary across animal-waste systems due to differences in manure moisture content and equipment use. The model simulates a stepwise cost function for manure-hauling/application cost, with cost coefficients defined by system type (lagoon, slurry, and dry) and distance interval hauled. Costs of manure incorporation into the soil (INC) are computed in equation 16 based on per-acre charge (C3), total onfarm and off-farm acres using manure, share of acres in cropland (SH_C) (as manure is not generally incorporated on pasture land), and share of manured cropland using incorporation (SH_I).

$$(17)\quad NM1_{ct} = (\,M\_TST + C\_NMP\,) * AFO_{ct}$$

$$(18)\quad NM2_{ct2} = S\_TST * (\,AC\_ONF_{ct2} + \sum_{ct} AC\_SPR_{ct,ct2}\,)$$

Selected nutrient management plan costs related to land application are identified for manure source farms and receiving farms. Equation 17 computes source-county costs (NM1), based on representative costs for manure testing (M_TST) and plan development costs (C_NMP), summed across confined animal-feeding operations (AFOs) in the source county. Equation 18 computes destination county costs (NM2) for soil testing, based on representative costs (S_TST) per acre of land receiving manure.

$$(19)\quad FS\_N_{ct2} = (\,PR_{N} * (N\_ONF_{ct,ct2,N} + \sum_{ct} N\_TRN_{ct,ct2,N}))$$

$$+ (PR_{P} * (P\_ONF_{ct,ct2,P} + \sum_{ct} P\_TRN_{ct,ct2,P}) * P\_PCT_{ct2})$$

$$+ (\,C\_AP * (AC\_ONF_{ct2} + \sum_{ct} AC\_SPR_{ct,ct2}))$$

Calculation procedures for fertilizer cost savings vary, depending on the nutrient standard in effect. In equation 19, savings calculated under an N standard (FS_N) include reduced chemical fertilizer purchases and reduced chemical application costs. Savings from reduced fertilizer purchases are computed based on the price (PR) of nitrogen (*N*) and phosphorus (*P*), and the quantity of manure nutrients used by crops. Nitrogen savings reflects the full application of manure-N, both onfarm (N_ONF) and off-farm (N_TRN), as all manure-N is assumed to be beneficially used in crop production since producers are assumed to meet nutrient management guidelines. Phosphorus savings reflects use of manure-P onfarm (P_ONF) and off-farm (P_TRN)—adjusted to capture that portion of P (P_PCT) that is beneficially used by the crop (or the ratio of applied manure at an annual P standard to applied manure under an N standard). Savings from reduced chemical application costs reflects the per acre cost of chemical application (C_AP) and total acres receiving manure under an N standard.[20]

$$(20)\quad FS\_P_{ct2} = (PR_N * (N\_ONF_{ct,ct2,N} + \sum_{ct} N\_TRN_{ct,ct2,N})) + (PR_P * (P\_ONF_{ct,ct2,P} + \sum_{ct} P\_TRN_{ct,ct2,P}))$$

In equation 20, savings calculated under an annual P standard (FS_P) reflect the value of only the manure-nutrient offset. There are no savings in chemical application costs (chemical fertilizer application is still required), as manure-N is insufficient to meet full crop needs. In contrast to the N standard where some portion of applied manure-P cannot be used by the crop, all applied manure nutrients are beneficially used under a P standard.[21]

$$(21)\quad FS_{ct2} = (FS\_N_{ct2} * SH\_N_{ct2}) + (FS\_P_{ct2} * (1 - SH\_N_{ct2}))$$

Equation 21 computes an acreage-weighted fertilizer cost savings (FS), based on the share of acres under an N standard and the more stringent (annual or multi-year) P standard.

[20] It is assumed that chemical nutrients are applied at crop-based rates, that manure nutrients directly offset nutrients obtained from chemical fertilizers, and that per acre field application costs are fixed regardless of the level of applied chemical fertilizer.

[21] The model can be easily modified to consider a multiple-year application of manure-P, in cases where soil-P levels allow for heavier manure applications. Under a multiyear P application, manure treatments are rotated over the farm acreage. Producers are permitted to apply multiple years' manure quantities at one time (up to the N standard level) on a given field where nutrients can be used in subsequent years of the multiyear rotation. Savings reflect the full value of the manure nutrients, as all applied manure nutrients are fully used by the crop. Savings also reflect a partial reduction in chemical field application costs, based on the share of acres treated annually within the multiyear rotation (equivalent to P_PCT). For the multiyear P case, equation 20 would be modified to include the additional cost savings:

$$+ (C\_AP * (AC\_ONF_{ct2} + \sum_{ct} AC\_SPR_{ct,ct2}) * P\_PCT_{ct2})$$