Appendix: Underfill and License on Demand

One can make some inferences about the level of underfill when License on Demand is employed to allocate quota rights. First, suppose that all trade is conducted in some standardized container load of X kilos. Call Z the tariff quota volume expressed in units of container loads: Z = QTRQ/X. The total amount of less-than-container-load shipments will equal N/2 full container loads where N is the number of applicants. This follows from the fact that the remainder is random and uniformly distributed over the range (0,1) with an expected value of 1/2. With N applicants, the sum of remainders is N/2.

Call the critical load value k. Because the remainder values are uniformly distributed, the expected value of remainders less than k is k/2, and the expected value of remainders greater than or equal to k is (1 - k) / 2. Similarly the number of remainders less than k is kN, and the number of remainders greater than or equal to k is (1 - k) N. The quantity of underfill is the number of loads less than k times the average quantity of such loads: $kN \cdot k/2 = \frac{1}{2}(Nk^2)$.

Number of remainders	N	Average quantity	1/2
Number of 0 loads	kN	Average quantity	k/2
Number of 1 loads	(1 - k)N	Average quantity	(1 - k)/2
Expected underfill	$\frac{1}{2}(Nk^2) = kN \cdot k/2$		

From the point of view of quota fill efficiency, the relevant magnitude is the expected underfill as a proportion of the quota. Define this value as $U = Nk^2/(2Z)$. Consider how changes in the components of this measure influence the value of U. An increase in k, the critical value of a shipment, increases underfill. The higher the critical value, the less likely a less-than-full shipment will be made. An increase in the number of applicants, N, also increases underfill. This follows because the number of applicants determines the number of less-than-full load remainders. Finally, increasing Z reduces underfill. This follows simply because Z is in the denominator, clearly with $2Nk^2$ invariant to Z, an increase in Z must reduce U.[1]

The value of Z could increase for two reasons. First, the quota could simply be expanded. Second, innovations in shipping can change Z even if Q^{TRQ} is unchanged. If, for example, smaller sized shipments become economically feasible, the number of kilos per shipment decreases. Thus, the number of shipments allowed by the quota increases: $Z = Q^{TRQ}/X$, reducing X increases Z. Finally, when the quota ceases to bind, $Q^{TRQ} > Q^*$ implies $Z > Z^*$, all license requests are granted, there are no remainders, and U collapses to zero.

[20]As long as Z > N, the value is invariant to the value of Z. When Z < N, the expected license quantity is less than one and the assumption of a uniform distribution of the remainder over the unit interval no longer holds. When Z < kN the average applicant will be granted a license for less than one critical load and therefore will not choose to ship. The lower the value of Z below kN, the more likely all applicants will choose not to ship and none of the quota will be used.