The Tornqvist Index as a True-Cost Index

The next step is to establish that a Tornqvist index can be interpreted as a true-cost index within the framework of Muellbauer's piglog model. First, note that the Tornqvist index is calculated from data about budget shares gathered in two different time periods that are multiplied by the log ratio of prices. We define the Tornqvist index for a household as:

$$I_t = 0.5 \, \Sigma_i \, (w_{ih1} + w_{ih0}) \ln (p_{i1}/p_{i0}), \qquad (20)$$

where w_{iht} is the budget share for the i^{th} good of the h^{th} household in period $t = 0{,}1$.

As noted above, Muellbauer's piglog cost function can be written as:

$$\ln c(u_h,p) = a(p) + b(p)u_h, \qquad (21)$$

where $a(p)$ and $b(p)$ are price functions and u_h is the level of utility of household h. The Hicksian budget shares of the piglog model for the h^{th} household in period t are:

$$w_{iht} = a_i(p_t) + b_i(p_t,u_{th}), \qquad (22)$$

where

$$a_i(p_t) = \delta a(p_t)/\delta \ln p_{it},$$
$$b_i(p_t) = \delta b(p_t)/\delta \ln p_{it}.$$

The true-cost index in period *1* relative to period *0* and referenced to utility u_{hR} is given by:

$$\ln P(p_1,p_0;u_{hR}) = \ln c(u_{hR},p_1) - 1\text{n } c(u_{hR},p_0). \qquad (23)$$

Fry and Pashardes (1989) showed that when the cost function is piglog, the Tornqvist index is the average of the true-cost indexes in $ln\ c(p_1,p_0,u_{H0})$ and $ln\ c(p_1,p_0,u_{H1})$, if $a(p)$ is quadratic and $b(p)$ is linear in log prices.

If we let:

$$a(p) = \alpha_0 + \Sigma\alpha_i \ln p_i + 0.5 \, \Sigma_i\Sigma_j \, \lambda_{ij} \ln p_i \ln p_j, \qquad (24)$$

and

$$b(p) = \beta_0 + \Sigma_i \, \beta_i \ln p_i, \qquad (25)$$

where α sums to one for all *i* and the λ and β sum to zero to satisfy adding up, homogeneity, and symmetry, then we can write the piglog cost function as:

$$\ln c(u_h,p) = g(u_h) + \Sigma_i \, (\alpha_i + \beta_i u_h + 0.5 \, \Sigma_j \, \lambda_{ij} \ln p_j) \ln p_i. \qquad (26)$$

The true-cost index in period 1 relative to period 0 would then be:

$$\ln P(p_1,p_0;u_{hR}) \, \Sigma(\alpha_i + \beta_i u_{hR})(\ln p_{i1} - \ln p_{i0}) \qquad (27)$$
$$+ 0.5 \, \Sigma_i \, \Sigma_j \, \lambda_{if} (\ln p_{i1} \ln p_{j1} - \ln p_{i0} \ln p_{j0}).$$

The budget shares at reference utility level R are:

$$w_{ihR} = \alpha_i + \Sigma_j \lambda_{if} \ln p_{jR} + \beta u_{hR}. \qquad (28)$$

Thus, the true-cost index can be written:

$$\begin{aligned}\ln P(p_1,p_0;u_{hR}) &= \Sigma_i\, w_{ihR}\,(\ln p_{i1} - \ln p_{i0}) \\ &- \Sigma_i\Sigma_j\, \lambda_{ij} \ln p_{jR}(\ln p_{i1} - \ln p_{i0}) \\ &+ 0.5\, \Sigma_i\Sigma_j\lambda_{ij}(\ln p_{j1} \ln p_{i1} - \ln p_{j0} \ln p_{i0}).\end{aligned} \tag{29}$$

If the true-cost index is calculated by setting R = 1, then setting R = 0, and lastly taking the average, all terms on the right-hand side except the first cancel out due to symmetry. Computing the average true-cost index for two periods by alternating the reference utility level between a base period and subsequent time period is, therefore, equivalent to computing the Tornqvist index.