Appendix A: Proofs

First, we prove that the decomposition in equation (9) is equivalent to the more traditional statement of the benefit formula in equation (4). Define total deductions (including unused deductions) as:

(A1) $D = D1 + D2 + D3 + D4 + D5 + D6.$

Substitute equations (A1), (5), (6), and (7) into equation (9) to get:

(A2) $B = M - 0.3 * G + 0.3 * D - 0.3 \text{ Max } (0, D\text{-}G) + E4$

$= M - 0.3 [G - D + \text{Max } (0, D - G)] + E4$

$= M - 0.3 [\text{Max } (G - D, 0)] + E4.$

Use equations (3) and (8) with (A2) to get:

(A3) $B = M - 0.3 * N2 + \text{Max } [0, 10 - (M - 0.3 * N2)]$

$= \text{Max } [M - 0.3 * N2, 10].$

This equals the right-hand side of equation (4) to complete the first proof.

Second, we prove that the shelter expense effect in equation (13) is equivalent to 0.3 times the shelter deduction (D5) in equation (2). From equations (10), (11), (12), and (13), we have:

(A4) $E2.6 = E2.6.1 + E2.6.2 + E2.6.3$

$= E2.6.1 + E2.6.2 + \text{Min } [0.3 * H^* - (E2.6.1 + E2.6.2), 0]$

$= \text{Min } [0.3 * H^*, E2.6.1 + E2.6.2]$

$= \text{Min } [0.3 * H^*, 0.3 * H - 0.3 * \text{Min } (0.5 * N1, H)]$

$= \text{Min } [0.3 * H^*, -0.3 \text{ Min } (0.5 * N1 - H, 0)]$

$= 0.3 \text{ Min } [H^*, \text{Max } (H - 0.5 * N1, 0)].$

This completes the second proof.