Theoretical Framework

Empirical analysis of the effect of the Food Stamp Program on household food consumption has typically involved estimating the relationship between household expenditure on food on the one hand and total money income and income from food stamps on the other. Thus, the procedure involves estimating a demand function:

$$y = y(J - s, s),$$

where y is the total household expenditure on food, J is the total household income from all sources, cash as well as coupons, and s is the value of food stamps received by the household. The total money income of the household is $(J - s)$. The cash-out puzzle is simply the following empirical observation. Estimates derived from samples consisting overwhelmingly of unconstrained households (i.e., households for whom $y > s$) seem to imply that the increase in household expenditure on food from one additional dollar's worth of food stamps is larger than that from one additional dollar of cash income. This in turn generates the following prediction about demand behavior by individual households. Suppose, at some given level of total income, J, and given some amount of food stamps, s, we observe that the household is unconstrained. Now suppose household food stamp income is changed such that total household income from all sources, J, is invariant. Then, at least for relatively small changes, the household will continue to remain unconstrained after the change. However, a relative decrease (increase) in the coupon component of household income will also lead to a fall (rise) in household expenditure on food. Note that there is no cash-out puzzle for constrained households; expenditure is expected to change under a cash-out for them. Thus, the puzzle may be formally defined as the following restriction on the household demand function for food.

For every $J > 0$, there exists a non-empty interval $(\underline{s}(J), \overline{s}(J)) \subset [0, J]$ such that: (a) for all $s \in (\underline{s}(J), \overline{s}(J))$, $y(J, s) > s$, and (b) y is an increasing function of s in this interval.

Our goal is to develop a model of household decision-making that generates demand behavior in accordance with this restriction.

In the food stamp literature, the assumption that multi-person households behave as if they are individual decisionmakers is ubiquitous. As the recent literature on intra-household decisionmaking shows, however, this assumption is actually quite questionable.[6] Intra-household distribution of resources may depend on the composition of total household income. Conversion of in-kind welfare income to cash income may simultaneously lead to a change in the intra-household division of resources. This, in turn, may lead to a multi-person household's exhibiting consumption behavior that cannot be explained in terms of the household's maximizing as an individual.

We now develop an alternative theoretical explanation of the cash-out puzzle along these lines. This explanation does not require the presence of any welfare stigma. We formulate our argument by means of a Cournot model of intra-household allocation.[7]

The Model

Assume a household with two adult members M and F.[8] Given any agent k, $k \in \{M, F\}$, we shall refer to the other agent as agent $-k$. Each agent k consumes a composite private good x_k. Each agent also derives utility from the total household purchase (and consumption) of food, y. Agent k's preference ordering defined over alternative combinations of household food purchase and the private good is represented by a strictly quasi-concave utility function $U^k(x_k, y)$.[9]

[6]See, for example, Alderman et al. (1955) for a survey.

[7]Earlier work in this tradition includes Ulph (1988), Woolley (1988), Lundberg and Pollack (1993), Kanbur (1995), and Dasgupta (1999).

[8]Generalization to a household with more than two members is straightforward.

[9]It is, of course, possible that an agent's preference ordering over alternative combinations of the private consumption good and total household purchase of food will depend on the intra-household division of food as well. We abstract from this complication. One simple way of arriving at our formulation through this route is to assume that intra-household distribution of food is determined according to some sharing rule in which each agent's food consumption depends only on total household availability of food. More complicated sharing rules, while compatible with our analysis, make the exposition cumbersome without adding anything substantive to the argument. Of course, the construction of these sharing rules is of interest to other analyses of the Food Stamp Program. Furthermore, estimating such sharing rules is nearly impossible since the data requirements would involve detailed information about which individuals make spending decisions for each good consumed by the household.

Purely for notational simplicity, we shall assume that the prices of all goods are unity. Household availability of food from food stamps is s, $s \geq 0$. Let the total income of the household from all sources, cash as well as food coupons, be J. Then the total cash income of the household is simply $(J - s)$. Each agent k has discretionary control over r^k amount of cash, $r^k \geq 0$. Clearly,

$$r^M + r^F = J - s.$$

Member k takes the other member's contribution to household food purchase, y_{-k}, and the availability of food from food stamps, s, as given, and chooses the allocation of his own discretionary cash income between food (y_k) and the private good, x_k. Let y be total household expenditure on food. By definition, we have:

$$\mathrm{y} = y_{-k} + y_k + s.$$

Thus, household food expenditure has the formal characteristic of a domestic public good, and agents play a Cournot game with respect to choice of contributions toward this domestic public good.[10] We assume that a Nash equilibrium exists in this game.[11]

The assumption of food as a domestic public good may appear troubling, since food is often invoked as an example of the classic alienable good. Note that, in our formulation, total household food consumption has the property of being a domestic public good only in a purely formal, and not necessarily substantive, sense. What we are essentially assuming is that each agent's preferences over alternative combinations of the private good and household food purchase is independent of how the total amount of food is distributed within the household (see footnote 9).

Given total household income, its division between cash and coupons, the division of discretionary control over household cash income among agents, and contribution toward household food purchases by the other agent, agent k's optimization problem is that of choosing the optimal levels of y and x_k, so as to maximize:

$$U^k(x_k, y)$$

subject to the budget constraint:

$$(1) \qquad r^k + y_{-k} + s = x_k + y,$$

and the additional constraint:

$$(2) \qquad y \geq y_{-k} + s.$$

The second constraint incorporates two restrictions. First, food stamps cannot be resold for cash.[12] Second, no agent can divert money allocated by the other agent for food purchase to his/her own private consumption.

Then, the solution to agent k's optimization problem, subject to the budget constraint (1) alone, yields the optimal levels of y and x_k as functions of total income from all sources, i.e., of $[r^k + s + y_{-k}]$. Let these unrestricted individual demand functions be given by:

$$\text{(i)} \qquad y = g^k(r^k + s + y_{-k}),$$

and

$$\text{(ii)} \qquad x_k = h^k(r^k + s + y_{-k}).$$

We impose the following restriction on unrestricted individual demand functions (and thus on individual preferences).

(A1): For all $k \in \{M, F\}$, g^k and h^k and are continuous and increasing in r^k.

The continuity assumption, while innocuous, is essentially made for convenience. (A1) merely requires that all goods be normal goods in the standard sense. This assumption suffices to ensure the uniqueness of the Nash equilibrium.[13] Then, the Nash equilibria yield single-valued household demand functions:

$$(3) \qquad x_k^N = X^k(r^k, J, s),$$

[10]The model can be made more realistic by allowing other public goods (for example, expenditure on children and housing) as well. This, while complicating the notation, however, does not add anything to the argument.

[11]See Bergstrom et al. (1986) for sufficiency conditions to ensure the existence of a Nash equilibrium.

[12]The no-resale restriction for food stamps is for convenience and can be relaxed to allow partial, but not complete, resale. Legally, food stamps cannot be sold for cash.

[13]See Bergstrom et al., 1986.

and

(4) $y^N = y^N_k + y^N_{-k} + s = Y(r^k, J, s).$

Since an agent can neither exchange any portion of the food provided through food stamps for cash nor divert the money contributed by the other agent for household food expenses to his own private consumption, it must be the case that in any Nash equilibrium for all $k \in \{M, F\}$,

(5) $Y(r^k, J, s) = \max[g^k(r^k + s + y^N_{-k}), s + y^N_{-k}],$

and

(6) $X^k(r^k, J, s) = \min[h^k(r^k + s + y^N_{-k}), r^k].$

An agent k is constrained in a Nash equilibrium if and only if, in that Nash equilibrium,

$[g^k(r^k + s + y^N_{-k}) < s + y^N_{-k}].$

Clearly, this is equivalent to the requirement:

$[h^k(r^k + s + y^N_{-k}) > r^k].$

Our next assumption is simply that all adult members of the household receive a share of any increase in cash income of the household.

(A2): For all $k \in \{M, F\}$, $r^k = r^k(J - s)$ and is continuous and increasing in its argument.

(A2), (3) and (4) together imply that the household demand functions generated by Nash equilibria can be rewritten as functions of total income, J, and food stamp income, s. Thus,

$x^N_k = x^k(J, s);$

and

$y^N = y(J, s).$

Our key assumption is the following:

(A3): Given any $J > 0$, there does not exist any $s \in (0, J)$ such that $[$for all $k \in \{M, F\}$, $[g^k(r^k(J - s) + s) = s]]$.

Suppose that the other agent spent his/her entire discretionary income on his/her own private good. Then, if the household received s amount of food stamps, the optimal amount of household food expenditure, from agent k's point of view, would be $g^k(r^k(J - s) + s)$. (A3) requires that this cannot be exactly equal to the amount of food stamps for both agents. This assumption introduces a minimal amount of heterogeneity in preferences and/or access to cash income between adult members of the household. To see how minimal such heterogeneity is, note first that (A3) is far weaker than the requirement that $g^k(r^k(J - s) + s)$ be different for the two agents at every possible level of food stamps. Note further that even the latter, stronger assumption (and hence (A3)) will be satisfied even if agents have identical preference orderings, so long as total household cash income is divided unequally. Conversely, even if household cash income is divided equally, the stronger assumption (and hence (A3)) will be satisfied if agents have different preference orderings. Of course, (A3) can also be generated by differences in both preferences and access to cash, combined in various ways.

Proposition 1. Suppose (A1), (A2), and (A3) are satisfied. Then, given any $J>0$, the household demand function for food $y(J, s)$ must satisfy the following:

(i) There exists $\bar{s}(J) \in (0, J)$ such that $[y(J, s) > s]$ for all $s \in [0, \bar{s}(J))$,

and

$[y(J, s) = s]$ for all $s \in [\bar{s}(J), J]$.

(ii) There exists $\underline{s}(J) \in [0, \bar{s}(J))$ such that $y(J, s)$ is an increasing function of s in the interval $(\underline{s}(J), \bar{s}(J))$.

Proof: See the appendix.

The intuition is simply that unconstrained households with a constrained individual will generate the cash-out puzzle. A numerical illustration is provided in the appendix with the proof of proposition 1.

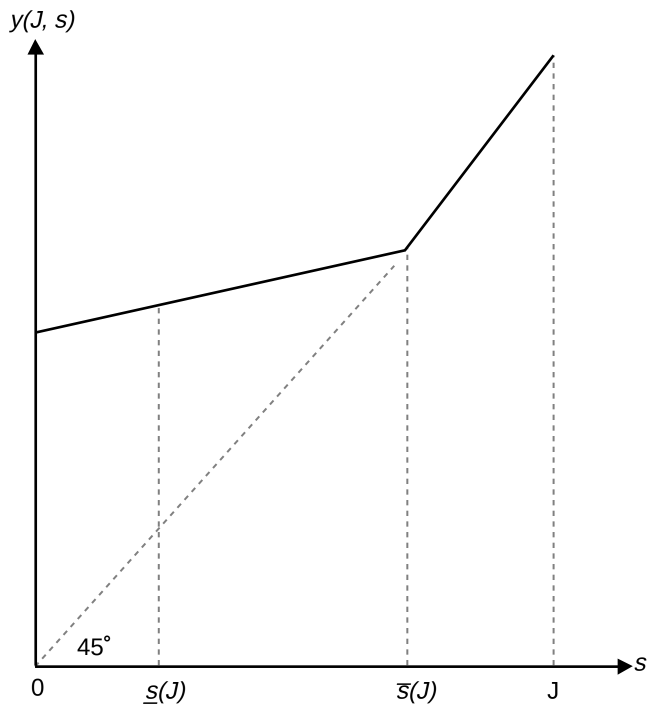
Suppose that a household has some arbitrary amount of total household income, J, consisting of cash income and food coupons. Clearly, different combinations of cash and coupons can generate J, the coupon component of such combinations lying in the interval [0, J]. Part (i) of proposition 1 states that, given (A1)-(A3), the household is unconstrained if and only if the amount of food stamps received by it is less than a particular positive number less than J, $\bar{s}(J)$. Part (ii) of proposition 1 implies that, given our assumptions, household demand behavior must necessarily exhibit the cash-out puzzle, as formalized above. There must necessarily exist a non-empty interval of food stamp values, ($\underline{s}(J)$, $\bar{s}(J)$), where the marginal propensity to consume food out of stamp income is larger than that out of money income, despite the fact that the household is unconstrained. In this interval, the larger the cash component in household income, the smaller the magnitude of household spending on food. Any substitution of cash income by food stamps in this region will necessarily increase household food expenditure, while leaving the household unconstrained. Note that it is possible that the demand function for food will be increasing in s throughout the interval [0, $\bar{s}(J)$). Figure 1a below shows how household food expenditure will change with changes in the coupon component of household income in this case.

Intuitively, the mechanism generating the cash-out puzzle in our model is the following. Given total household income, (A1), (A2), and (A3) together imply that there will necessarily exist a region of values of food stamps, ($\underline{s}(J)$, $\bar{s}(J)$), such that, if the actual amount of food stamps received by the household falls in this region, then one agent will be constrained. Furthermore, the other agent will necessarily be unconstrained. The unconstrained agent will contribute a positive amount toward household food expenses. Consequently, total household food purchase will be greater than the amount of food stamps, i.e., the household will be unconstrained. Now, consider a relative increase in household cash income due to a cashing-out of food stamps. This makes a larger amount of cash available to the constrained member, allowing that member to increase his/her nonfood expenditure. So long as the post cash-out amount of food stamps remains within the interval ($\underline{s}(J)$, $\bar{s}(J)$), the constrained agent will continue to stay constrained, preferring to spend all additional cash income on his/her private good. The unconstrained agent will stay unconstrained; consequently, the household will stay unconstrained as well. The unconstrained member of the household will increase his/her cash contribution toward household food purchase to compensate for the reduction due to the fall in the stamp component. However, the conversion effectively reduces the total income (cash and coupons) available to this agent. Since household food expenditure is a normal good, this causes the unconstrained agent to increase his/her cash contribution by less than the magnitude of the reduction in food stamps. Consequently, the total food purchase falls. The exact opposite happens when cash income is converted to food stamps.

If the coupon component is increased to $\bar{s}(J)$ or beyond, both agents and, therefore, the household will become constrained. On the other hand, it is possible, but not necessary, that a large reduction in the coupon component of household income beyond $\underline{s}$ will make the in-kind constraint slack for the agent for whom it was binding earlier. In that case, both agents will become unconstrained. It can be easily shown that further conversions of coupon to cash will leave household demand invariant. This case is depicted in figure 1b. The marginal propensity to consume food out of cash income is exactly the same as that out of food stamps in the interval [0, $\hat{s}(J)$).

The critical assumption driving our results is (A3). If this restriction is violated, then given (A1) and (A2),

Figure 1a.
(A1) - (A3) Hold: Cash-out puzzle



the following can be easily established. There will necessarily exist some value of food stamps between zero and *J*, say $\tilde{s}(J)$, such that both agents will be constrained when the actual amount of food stamps received by the household is greater than $\tilde{s}(J)$. Both agents will be unconstrained when it is less. Household demand behavior will be exactly as predicted by the Southworth model. This is depicted in figure 2. Note that the same conclusions can be generated in essentially the same way by modeling the intra-household allocation process as a Stackelberg game.

The model developed in this section implies that, if the proportion of unconstrained multi-adult households with constrained individuals is significant in a sample, then estimates derived from this sample will yield a marginal propensity to consume food out of food stamps significantly larger than that out of cash income. To seek empirical confirmation for the model, we, therefore, need to check whether the cash-out puzzle in the data largely arises from consumption behavior of multi-adult households.

In line with the standard practice in the literature, we have treated cash income from different sources equivalently in our model. This was done purely for simplicity of exposition. The model can be easily generalized to allow different intra-household sharing rules for cash income arising from different sources. Thus, for example, one may assume that cash welfare payments and cash labor income are shared differently. It is intuitively plausible that if households start getting cash instead of stamps, members may collectively decide, perhaps due to inertia, to make only part of that additional cash available for discretionary spending, while continuing to earmark the rest for non-discretionary expenditure on food as before, at least initially. This can be captured in our model at the cost of some increase in notational complexity by the assumption that a welfare check scheme increases each member's discretionary income by an amount less than that when cash-out takes the form of an increase in household non-welfare cash income. It should be intuitively clear that this version would predict a larger propensity to consume food out of welfare checks than out of income. In general, there is no strong *a priori* reason to assume that the intra-household distribution of cash would be independent of the composition of the cash-flow with regard to its source. Our framework is thus consistent with differing marginal propensities to consume food for cash from different sources. On

Figure 1b.
(A1) - (A3) Hold: Cash-out puzzle

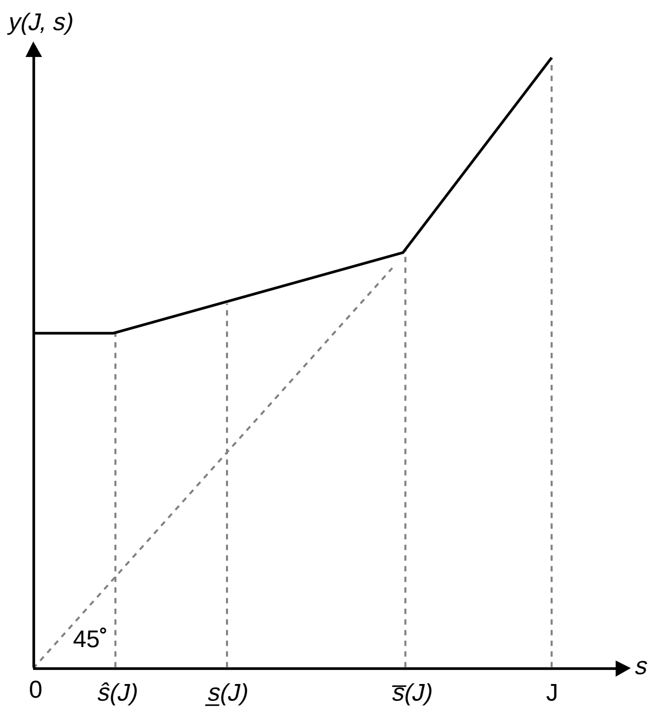
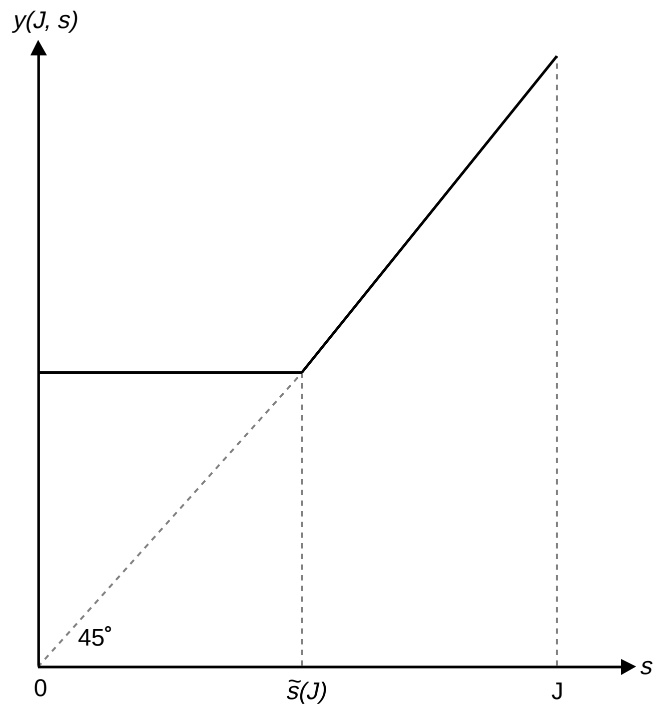


Figure 2.
(A1) and (A2) Hold, (A3) violated: No cash-out puzzle



the other hand, our model would be decisively refuted if it can be shown that the marginal propensity to consume food out of cash income, whether from welfare payments or otherwise, is higher than that out of food stamps.

Analysts have conjectured that while food stamps are not targeted toward women *per se*, since women are the main food purchasers, the delivery mechanism creates an entitlement (in the sense of a right-of-control acknowledged by other household members) to such transfers, unlike cash transfers (Alderman et al., 1997, p. 278).[14] To the extent that this suggestion implies that a significant proportion of cash welfare transfers may be controlled by men, an assumption formalized in (A2) earlier in this section, it is intuitively plausible. It is, however, difficult to see what the notion of entitlement (i.e., effective control) implies in this context, unless one also assumes resale possibility for food coupons. Otherwise, since *de facto* property rights over food coupons can be exercised only through purchase of food, in terms of household food expenditure, it does not matter which member has such property rights (though this may influence its composition). Note that in our model, the cash-out puzzle arises independently of any assumption about the intra-household division of control over food stamps. Note also that the conjecture that men control the entire amount of any cash welfare transfer by itself does not explain the cash-out puzzle, unless it is additionally assumed that men do not contribute to the domestic purchase of food. Indeed, it is intuitively clear and can be shown formally that, with identical preferences, women will be sole contributors to household food purchases only if they earn significantly more than men. If, instead, women earn significantly less, they may not contribute any part of their personal income toward food purchases, choosing to use only the money allocated by men and the available amount of food stamps, even if they are solely responsible for the actual shopping and preparation of food. In that case, if men control the entire amount of any cash welfare transfer, a cash-out will in fact leave household purchase of food invariant.

[14]"Entitlement" is sometimes used to describe programs from which any eligible recipient is entitled to receive benefits. The Food Stamp Program is one such program while, for example, housing assistance programs are rationed such that some eligible households cannot receive benefits. The meaning of "entitlement" here is quite different, and we trust that the reader will not confuse the two definitions.