Appendix E: The Social Accounting Matrix (SAM) Multiplier Framework

For the regional (and national) economies, a social accounting matrix (SAM) presents a snapshot of a regional (or national) economic equilibrium. It is the accounting framework in matrix form that underlies the elaborate circular flow diagrams of economic activity found in basic economics texts. The strength of the SAM is the integration of the input-output table with a set of household, government, capital, rest-of-the-U.S. (ROUS), and rest-of-the-world (ROW) accounts in order to represent the complete set of revenue and income flows between production, income, consumption, investment, and trade.

As a double-entry accounting framework of debits (expenditures) and credits (receipts), the column sum of expenditures made by each account is equal to the row sum of its receipts. For the firm accounts (in Figure E.1), total costs is the column sum of purchases of intermediate goods and services from other firms (**A**), wages paid to labor and profits paid for services rendered by owners of financial and real property assets (**F**), indirect business taxes ($\mathbf{T_{IB}}$), and purchases of imports (**M**). Firms' total costs equal the row sum of total sales of their output made to other firms (**A**), households (**C**), government (**G**), investment purchases of capital goods by businesses and government (**I**), and exports outside of the region or outside of the country (**E**). Total factor income paid by businesses (**F**) is redistributed to households (**Y**), to government as social security payments and taxes on profits ($\mathbf{T_F}$), and to the capital account as business savings in the form of depreciation and retained corporate profits ($\mathbf{S_B}$). For households, the column sum of total expenditures on consumption goods and services (**C**), taxes ($\mathbf{T_H}$), and savings ($\mathbf{S_H}$) equals the row sum of total income received in the form of wages and property income (**Y**), remittances from other enterprises

Figure E.1
The social accounting matrix

Account:	Production	Factors	Households	Other institutions	Total
1. Production	A		C	I, G, E	Total sales
2. Factors	F				Factor income
3. Households		Y	R	G_T	Household income
4. Other exogenous institutions	T_{IB}, M	T_F, S_B	T_H, S_H	S_G, G_G, S_F	Savings, tax revenue, imports
Total	Total costs	Factor income	Household expenditures	Investment, government outlays, exports & foreign savings	

Source: Economic Research Service, USDA.

and households (**R**), and government transfers (**G_T**). For State/local and Federal governments, the row sum total of tax receipts from all sources (**T_{IB}, T_F**, and **T_H**) is equal to the column sum total of government expenditures on goods and services, government transfers to households and firms (**G_T**), transfers among the different levels of government (**G_G**), and any budget savings (**S_G**). For the capital account, investment purchases (**I**) equals the row sum total of savings from all sources (**S_B, S_H, S_G**, and **S_F**). Finally, equilibrium in the ROUS and ROW accounts means that the row sum of imports purchases (**M**) is equal to the column sum of exports out of the region (**E**) plus capital inflows or "foreign savings" (**S_F**).

The SAM framework possesses an extraordinary flexibility enabling us to tailor the dimensions of the SAM to the problem at hand. Our SAMs focus on those sectors most affected by CRP enrollment changes: agriculture, spending on outdoor recreation, and household expenditures out of transfer income. For our regional SAMs, we aggregate the 478 sectors in the IMPLAN database into 12 industrial accounts (13 accounts for the national SAM). These SAMs have five agricultural sectors: livestock, grains, oilseeds, hay & pasture crops, and other crops. For the Southern Plains, cotton replaces oilseeds as a sector; for the national SAM, the cotton sector is added to the set of agricultural sectors. The seven nonagricultural sectors are agricultural inputs, food processing, industry, wholesale trade and transportation, retail trade, eating and lodging establishments, and services. The "industry" sector itself is an aggregation of manufacturing, mining, energy, and utilities sectors. Outdoor recreation expenditures are broken down into expenditures on eating and lodging, retail trade (e.g., household purchases of equipment and supplies), and services (e.g., household expenditures on permits, fees, guide services).

In our SAMs, we aggregate the nine income classes of households in the 1996 IMPLAN database into three classes. "Low income" or poor households receive less than $20,000 in income from all sources. This income cutoff serves as a good aggregate approximation for households living below the poverty threshold. "Middle income" households receive between $20,000 and $70,000 in income. "High income" households receive income greater than $70,000. The purpose for this household disaggregation in our SAMs is to quantify the different expenditure patterns exhibited by each household group. Low-income households spend more out of every $1 of income on consumption goods than do middle- and high-income households. Thus, income transfers targeted to different household groups will yield distinct results by household class on consumption, taxes, and savings.

The national model, together with models for each of the three multicounty areas we study, require four separate SAMs. In addition, in order to determine the urban-rural distribution of economic impacts within each region, we also construct rural SAMs for the Northern Plains Crescent and the Southern Plains Ellipse regions. These rural SAMs exclude the metropolitan counties located in each region. In this way we can assess the extent to which direct impacts occurring in these regions' rural areas also generate indirect impacts on output and jobs in their urban areas.

The SAM Multiplier Model

The SAM also serves as the basic building block for the SAM multiplier model. We use the SAM multiplier model to assess the direct and indirect impacts of abolishing CRP funding on the national economy and the three regional economies. The SAM multiplier model completely captures the interlinkages among revenue, income, and expenditure flows made by households and firms. The matrix multiplier obtained from the SAM captures not only the direct and indirect effects in production but, also induced effects. In production, direct effects represent the initial impacts of an outside shock on a particular sector. Indirect effects refer to a particular sector's demands for intermediate goods. Induced effects refer to those demands for goods and services made by households spending their new income derived producing new output induced by the outside shock. In addition and even more importantly, the SAM multiplier also captures the direct and indirect effects associated with exogenous shocks to households. In figure E.1, the submatrix **A** contains just the intermediate purchases among firms that are characterized by input-output multipliers, whereas the dotted rectangle contains *all* of the endogenous flows among households, factors, and firms embedded in the SAM multipliers. At the same time, the SAM multipliers account for the leakages and injections occurring at their proper entry points in the circular flow.[91]

To obtain the SAM multiplier matrix, we begin by transforming the SAM as a 23x23 matrix of expenditure shares $\mathbf{\Gamma}$. The elements in each column of $\mathbf{\Gamma}$ sum to 1. The i^{th} column in $\mathbf{\Gamma}$ represents the percent of account i's outlays accruing to each of the other accounts in the SAM. Since the elements of a 1x23 vector of column totals ($\mathbf{y}$) and a 23x1 vector of row totals ($\mathbf{x}$) of the SAM accounts are equal, we can express the SAM as,

$$\mathbf{\Gamma} \cdot \mathbf{x} = \mathbf{y}' \qquad (1)$$

Given our shares matrix $\mathbf{\Gamma}$, let **B** be the matrix of the subset of these coefficients comprising the endogenous accounts contained in the dashed rectangle in figure E.1: production activities, factors, and households. The exogenous, government, capital, and the rest-of-the-world accounts are excluded. We express a condition for an accounting equilibrium as the vector of total output and income flows ($\mathbf{z}$) that supports the vector sum of endogenous household and firm demands ($\mathbf{B} \cdot \mathbf{z}$) plus the vector of row sums of exogenous demands ($\mathbf{w}$),

$$\mathbf{z} = \mathbf{B} \cdot \mathbf{z} + \mathbf{w} \qquad (2)$$

Note that **z** is a subset of row totals **x** for the entire SAM corresponding to the endogenous accounts defined in **B**. The vector **w** is the subset of row sums of the exogenous demands placed on the endogenous accounts defined in **B**; it does not include exogenous flows among the exogenous accounts themselves. In equilibrium, the SAM multiplier is easily obtained,

$$\mathbf{z} = (\mathbf{I} - \mathbf{B})^{-1} \cdot \mathbf{w} = \mathbf{M} \cdot \mathbf{w} \qquad (3)$$

where M = $[m_{ij}]$. Each sectoral multiplier, m_{ij}, represents the induced income flow to account i for services performed for account j, as a result of

[91] For example, social security payments are treated as taxes on factor income, not as household income. Hence, this leakage is subtracted from the flow of factor income disbursed to households. The SAM framework treats factor income paid outside of the region as a leakage from factor income, not household income. Accounting for these leakages out of factor income explains why household income multipliers differ from factor income multipliers.

one unit of exogenous expenditure placed on (or one unit of exogenous income transferred to) sector j. The j^{th} column vector of multipliers $\mathbf{M_j}$ captures the impacts of an exogenous shock to the j^{th} account on all endogenous accounts in the SAM. The diagonal multiplier m_{jj} measures the direct impact of the shock to the initial sector j. The other off-diagonal multipliers m_{ij} represent the indirect impacts of the shock affecting the other industries, the returns to factors, and household incomes by type of household. Keeping in mind the graph of the circular flow of economic activity found in basic economics texts, the SAM multiplier model is able to account for the effects of an exogenous shock at all the different points in the endogenous circular flow, regardless of whether the shock first affects a particular firm, factor, or household (or set of firms, factor, or households).

Sectoral labor requirements for the production activities in the SAM are calculated as

$$\mathbf{l = L \cdot (M_A \cdot w)} \tag{4}$$

where $\mathbf{l}$ is the Hadamard product of $\mathbf{L}$, the vector of sectoral labor/output ratios, and ($\mathbf{M_A \cdot w}$), the vector of sector outputs supporting equilibrium in equation (3).[92] Elements in $\mathbf{L}$ are expressed as the number of jobs required to produce \$1 million worth of output for each production activity in $\mathbf{A}$; $\mathbf{M_A}$ is the 13x19 submatrix of interindustry, factor income, and household expenditure multipliers in $\mathbf{M}$ that affect 13 production activities.[93, 94]

Given the accounting equilibrium in equations (3) and (4), equations (3′) and (4′) express the endogenous responses to the exogenous shock $\Delta\mathbf{w}$: [95]

$$\Delta\mathbf{z} = \mathbf{M} \cdot \Delta\mathbf{w} \tag{3′}$$

$$\Delta\mathbf{l} = \mathbf{L} \cdot (\mathbf{M_A} \cdot \Delta\mathbf{w}) \tag{4′}$$

Strictly speaking, $\Delta\mathbf{l}$ represents the induced changes in labor demand. Although these simulations project increases or decreases in labor demand, *ex post* changes in actual employment levels cannot be assessed by this framework.

The SAM multiplier model in equations (1)-(4′) represents the most general case of fixed-coefficient, linear multiplier models. It is considered as the benchmark multiplier model. Extended input-output models (such as Type II, Type III, and Miyazawa multiplier models) represent partial closures of multisectoral equilibrium (Pyatt, 2001). These latter models either do not capture the full impacts, or, depending on the parameters used, produce approximations that may understate or overstate the impacts (Holland and Wyeth, 1993).

Finally, the results from our simulations must be interpreted with some caution. As a member of the family of fixed-coefficient linear multiplier models, the SAM model assumes that the supply response is perfectly elastic. This assumption describes an economic environment without scarcity. That is, there always exist unemployed resources sufficient to meet the new demands projected by our simulations. Moreover, if there exists an input supply bottleneck in the regional economy, then the industry is

[92] If $\mathbf{A} = (a_{ij})$ and $\mathbf{B} = (b_{ij})$ are each mxn matrices, their Hadamard product is the mxn matrix of elementwise products $\mathbf{A \cdot B} = (a_{ij}b_{ij})$.

[93] The dimensions of $\mathbf{M}$ are 19x19 for the national SAM and 18x18 for each of the three regional SAMs.

[94] The SAM production multipliers include induced effects and, therefore, are larger than Leontief input-output multipliers. One can use the Pyatt-Round decomposition to recover the input-output multipliers from the SAM production multipliers (Pyatt and Round, 1979).

[95] The vector $\Delta\mathbf{w}$ could easily represent a matrix in which each column $\Delta_{\mathbf{wj}}$ represents a single scenario. In this case, each column of the matrices $\Delta\mathbf{z}$ and $\Delta\mathbf{l}$ represents the results of a single simulation.

assumed to be able to costlessly import the good from outside the region. This assumption means that we interpret our output and job estimates represent at best as upper bounds of a positive endogenous response and lower bounds of a negative endogenous response.[96] In an economywide framework that allows for scarce inputs to flow to their most profitable uses, the estimates of the impacts of abolishing the CRP would be lower.

Modeling Urban-Rural Differences

We decompose the regional results into impacts on the urban economy versus the rural economy for the Northern Plains Crescent and the Southern Ellipse. Given our regional SAM models, we construct "rural" SAM models that just include the nonmetro counties embedded the regional models and exclude the metro counties. Then, we analyze the effects of removing the CRP using these two rural models. The differences in employment and output between the two models represent the urban impacts of removing the CRP in these two regional economies. We use the job/output coefficents for the regional and rural models to construct the job leakage statistics reported in Figure 4.3 (see Vogel, 2003).

[96] Another assumption that prices do not change in response to an exogenous shock is quite appropriate for the models of the regional economies, since prices are determined outside their borders. The fixed-price assumption does not cause undue harm to simulations using the national model, since the size of the CRP shock is less than 0.01 of 1 percent of aggregate national income. If the shock were larger, the policy analyst would anticipate larger price changes creating larger effects in output and factor markets that could not be captured in this framework.