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Documentation for the Agri-Food Economic Data System (Ag-FEDS): A More Complete Accounting of the U.S. Agri-Food Economy

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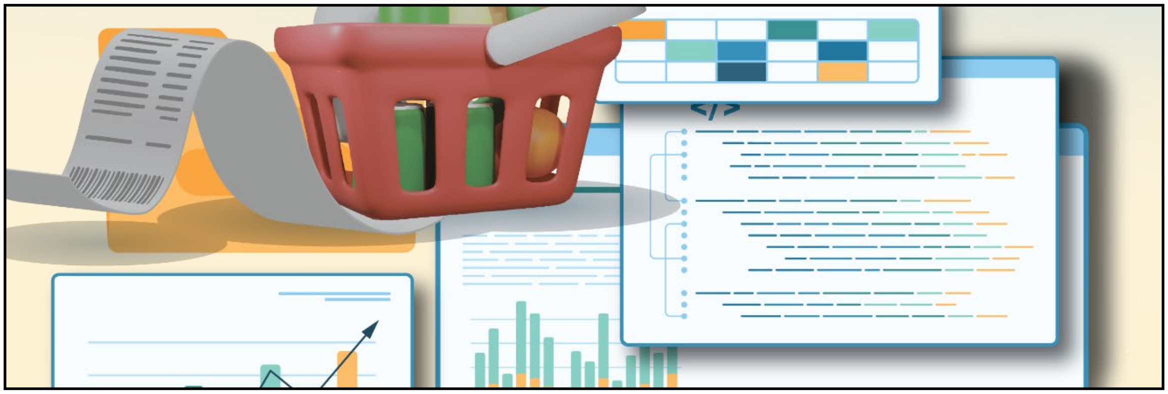

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Abstract

To expand the scope and uses of USDA, Economic Research Service's (ERS) data products and research on the food economy and markets, USDA, ERS researchers have developed a new data product known as the Agri-Food Economic Data System (Ag-FEDS). Ag-FEDS is an integrated system of economic data that elaborates the linkages between all production and consumption activities throughout the economy. This data system improves the clarity and accuracy of agricultural and food economy data when measuring how all production is distributed among consumers, businesses, governments, and global nations. This report introduces Ag-FEDS and shares details of the models and assumptions supporting the new tool. USDA, ERS has identified official U.S. Government data and applied statistical and economic modeling best practices to produce the most complete accounting of the U.S. food economy to date. In comparison to the accounts currently used for the USDA, ERS Food Dollar Series data product, Ag-FEDS more than doubles activity and commodity coverage (from 153 to over 350 activities and commodities) and captures up to 11 percent more annual food expenditures not included in conventional food Gross Domestic Product measures. Further, Ag-FEDS disentangles and measures over 40 distinct food commodity and marketing channel supply chains and applies accounting and modeling refinements that produce more accurate measures of the activity sequences, culminating in annual food and beverage purchases. To facilitate replication, this report describes primary data sources and every calculation, written entirely in matrix algebra.

Keywords: Agri-food value chains, food economy, social accounting matrix (SAM), structural modeling

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Chapter 1: Introduction to the Agri-Food Economic Data System

Summary for Chapter 1

Introduction

The overarching goal in compiling the Agri-Food Economic Data System (Ag-FEDS) accounts is to provide a more complete and accurate accounting of food and beverage costs and resource use along the sequence of activities from farm production through points of purchase. With more than 10 years of experience in developing model derived economic statistics on the U.S. food economy, the U.S. Department of Agriculture (USDA), Economic Research Service (ERS) has developed a more informed understanding of modeling best practices tailored to the specific requirements of targeted economic statistics. Use of the Food Dollar method at USDA, ERS has expanded and evolved since its introduction in 2011 (Canning et al., 2017; Canning & Stacy, 2019; Canning et al., 2020; Hitaj et al., 2019; Rehkamp & Canning, 2018; Yi et al., 2023). USDA, ERS' work has uncovered several opportunities to expand the scope and sharpen data and models' focus on the structure and organization of the U.S. food economy. To implement these developments, we expanded the source data to compile timely and detailed supply tables, use tables, and personal consumption expenditure (PCE) accounts that have been used to build the Food Dollar model dataset (Canning, 2011) and its application to other input-output (Canning et al., 2022) and social accounting matrix (SAM) multiplier models (Canning & Stacy, 2019).

In addition, we harmonized these expanded data sources with other USDA, ERS data products to explicitly account for the hidden food economy as well as transactions that get obscured by the aggregated nature of the national accounts. We employed well established best accounting and modeling practices that correct problems caused by the multiproduct accounting issues and limitations of conventional multiplier analysis. These combined changes from current practices at USDA, ERS were substantial. This report describes primary data sources that were used to develop Ag-FEDS and sequentially reports every calculation (in matrix algebra) used to compile our target data system.

Ag-FEDS is an integrated system of economic data that elaborates the linkages between all production and consumption activities throughout the economy. The data in Ag-FEDS were organized into a social accounting matrix, or a structured data table that can be accessed as a text based file that stores data in a tabular format. Ag-FEDS data has improved the clarity and accuracy of agricultural and food economy data when measuring how all production is distributed among consumers, businesses, governments, and global nations. The Ag-FEDS accounts described in this report updated and improved the data currently underlying USDA, ERS' Food Dollar data product (Canning, 2011), Supplemental Nutrition Assistance Program (SNAP) multiplier model (Canning & Stacy, 2019), and resource use models (Canning et al., 2022). Additionally, the Ag-FEDS data could also be used within a computable general equilibrium (CGE) framework to analyze consumer and producer responses to policy interventions and other events. The updates and improvements documented in this report can be applied to international applications of the food dollar data methods (e.g., Canning et al., 2016; Santeramo et al., 2024; Yi, et al., 2021).

1.1 Innovations of the New Approach

To compile regular and reliable data with sufficient detail for the types of research applications discussed above, we worked with both annual and benchmark years for the U.S. Department of Commerce, Bureau of Economic Analysis' (BEA) accounts. BEA's benchmark years occur every 5 years corresponding to the years covered by the Economic Census (U.S. Department of Commerce, Bureau of the Census) and Census of Agriculture (USDA's National Agricultural Statistics Service), and our analysis dates back to 1997. For our first innovation, we developed an optimal aggregation of BEA's Detail, and Summary Underlying Detail Supply, Use, and PCE-bridge tables and satellite underlying detail tables from BEA and the U.S. Department of Commerce, Bureau of the Census (Census Bureau). We applied constrained maximum likelihood (CML) models to estimate our target multiplier model dataset. Using several standard diagnostic tests, we provided a compelling validation for the CML estimation model across all subaccounts and over the entire time series. Beyond these accuracy measures of the CML results, it is important to note that all relevant annual and census year BEA data products used in this series were exactly replicated in the target model dataset.

Secondly, to address the issues of commodity flows being locked into the fixed production technologies, a model assumption, we employed an accounting technique used by BEA, which they refer to as redefinitions. This involves the allocation of secondary products and their associated inputs within both make and use tables such that they are reassigned to the industry in which they are the primary products (BEA, 2011). For example, when the airline industry provides foodservice to their customers, we identify the specific purchased and supplied inputs used by the airline industry to supply foodservice, and reclassify these services as outputs of the foodservice industry and deduct this as an output of the airline industry. Our approach to redefinition was to employ a simple mathematical programing model that extracted the foodservice inputs including primary factors from the nonfoodservice activities.

To further untether food commodity flows locked into the fixed production technologies, we next introduced a voucher accounting subsystem. Our voucher subsystem of accounts was classified into five categories: (1) meals as a business expense of employers/proprietors, (2) food and beverages furnished at work, (3) meals at schools and colleges, (4) meals provided at other institutions,[1] and (5) overhead costs for voucher redemptions. Overall acquisitions through these channels represented a market value of several hundred billion dollars in 2017 (for more information, see BEA's *Use Table, Before Redefinitions, Producers' Value, 2017* report (U.S. Department of Commerce (USDOC), Bureau of Economic Analysis (BEA), 2024c)).

We also found total food-at-home spending to be $91 billion higher in 2017 according to BEA (USDOC, BEA, 2024d) relative to USDA, ERS measures (USDA, Economic Research Service (ERS), 2023a). Over the 1997–2022 period, BEA measures ranged between 9–14 percent higher. This amount was uncovered and we refer to as the "hidden food economy." We identified passthrough accounting procedures applied to retail activities as an important source of these discrepancies for food-at-home spending. In our voucher accounting approach, we also identified foodservice as an intermediate expense of production activities, representing business related expensing of meals, and this represents an important source of discrepancies between USDA, ERS and BEA measures of food spending for foodservices as our fourth innovation. From a national income accounting perspective, business expenditures on foodservices have contributed to total Gross Domestic Product (GDP) through the commodities these businesses produce. USDA, ERS measures these costs as food expenditures. This difference in accounting practices, along with the passthrough accounting already discussed, appears to fully explain the significant discrepancy in foodservice expenditures measured by BEA and USDA, ERS. Other definitional differences between USDA, ERS and BEA measures of various institutional activities involving food and foodservices

[1] These include hospitals, nursing homes, prisons, and both public and private food assistance.

appeared to be fully explained by the untethering process among food, beverages, and foodservices such that many of the untethered commodities will be redirected to numerous voucher activities. We leveraged the USDA, ERS Food Expenditure Series data product and other USDA, ERS data on price spreads to reconcile and fully account for every discrepancy between USDA, ERS and BEA statistics.

Finally, researchers can routinely apply several finetuning techniques to better align multiplier model calculations with the intended measures of the USDA, ERS Food Dollar Series data product. These finetuning techniques include the following: (1) addressing negative value entries (with a few notable exceptions) by reversing direction of flows; (2) measuring and addressing deficits in food wholesale and retail services that facilitate all food dollar expenditures; and (3) integrating the Supply and Use tables and PCE bridge tables into a single expanded account to increase the number of commodity and marketing channel food dollar statistical series that can be annually reported. For more information about these accounting issues, see chapter 3 of this report.

Chapter 2: An Overview of the Social Accounting Matrix and its Relationship to the U.S. System of National Accounts and Input-Output Accounting

Summary for Chapter 2

Introduction

The Agri-Food Economic Data System (Ag-FEDS) will be compiled as a social accounting matrix (SAM) account and all structural modeling applications using Ag-FEDS will be either SAM multiplier models or computable general equilibrium (CGE) models. A SAM account (Stone & Brown, 1962; Pyatt & Rounds, 1979; Canning & Stacy, 2019) organizes and describes transactions throughout an economy to get a complete, numerical picture of economic activity. A SAM account is an extension of the input-output account. The basic production unit in an input-output account is called either an industry or commodity depending on which definition is used[2] and each producer makes only one output.[3] The basic SAM account describes production of one or more commodities by each activity and this multioutput feature of activities is explicitly measured in the SAM account.

For the United States, the basic building blocks of both input-output accounts and SAM accounts are the national make and use tables that are part of the U.S. system of national accounts (SNA) compiled by the U.S. Department of Commerce, Bureau of Economic Analysis (BEA) (USDOC, BEA, 2009). Additional building blocks for a basic SAM account include several tables from the BEA National Income and Product (NIPA) accounts (USDOC, BEA, 2024f). The SAM account has four main partitions, each having one or more subaccounts. The main partitions include production, consumption, investment, and international accounts. The circular financial flows throughout a SAM account are depicted in figure 1. The flows represent payments recorded between the subaccounts labeled in figure 1. These flows are circular since current sales in final markets (e.g., consumption, investment, and export/import markets) finances future production for these markets, and past production supplies current final market sales, in a recurring cycle. Specific transactions within the SAM account are described in table 1.

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[2] BEA published industry-by-industry and commodity-by-commodity input-output tables wherein industry and commodity are the production units (USDOC, BEA, 2009).

[3] Input-output production technologies are compiled from make and use tables as weighted averages of one or more industry production technology, as described in BEA's latest input-output manual (USDOC, BEA, 2009) by equation 12 for commodity-by-commodity input-output accounts or equation 14 for industry-by-industry input-output accounts.

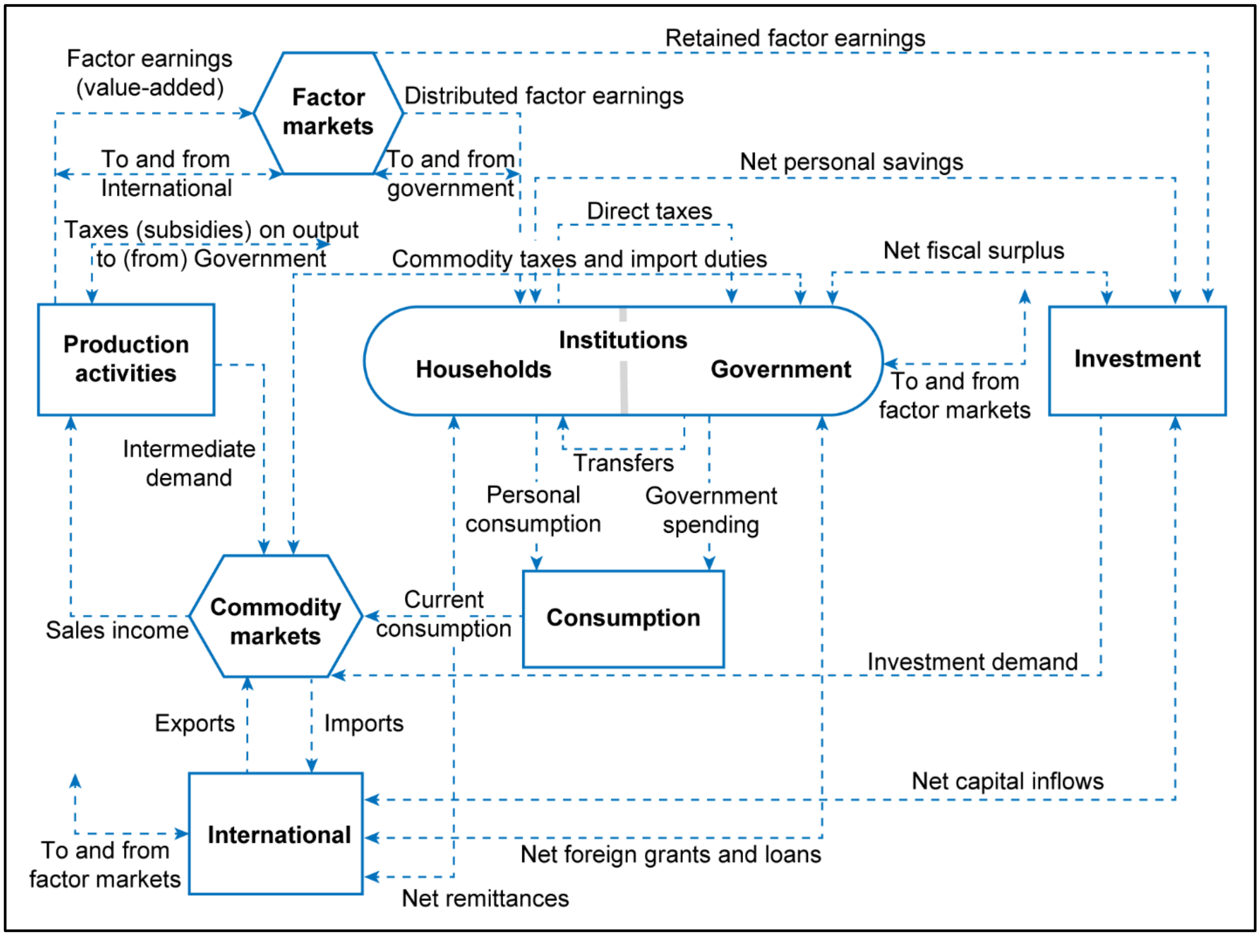
The production account has three subaccounts including: (1) an activity (*A*) account, (2) a commodity (*C*) account, and (3) a primary factor (*P*) account. All domestic production was measured in the activity account, which was comprised of several distinct activity groups. These three activity groups depicted in the basic SAM schematic in table 1 include agriculture, manufacturing, and other. In table 1, each activity contributes to production of one or more commodities, as described at the intersection of *A* rows and *C* columns (i.e., farm commodities, goods, and services). To produce commodities, each activity employs the services of primary production factors such as labor, land, buildings, equipment, and other property (table 1). For example, tomato farming requires labor to run the mechanical tomato harvesters and an outbuilding to store the tomatoes. Although we presented a single row and column for the primary factor account, this account can have many elements to represent several primary factors such as different categories of labor and capital. Each activity employed services from their primary factors in combination with intermediate commodity input use produced by its own and/or other production activities and/or imported from international sources (table 1). The activities may be subject to indirect taxes (e.g., output, corporate and excise taxes, and fees less subsidies) by Federal and State governments. These financial flows are depicted by the three outbound arrows[4] from the "Production activities" box in figure 1.

Commodity sales include intermediate demand for domestic production activities (e.g., tomatoes purchased for the fruit and vegetable canning activity), institutional sales to meet household and government consumption demand and to meet investment demand (e.g., tomato sauce purchased to serve in schools provided by the National School Lunch Program), and export sales to meet international consumption and investment demand (e.g., tomato sauce exported to other countries or stored for future sales) (table 1). All proceeds from commodity sales have multiple claimants (figure 1). Claims on proceeds of commodity sales include government claims in the form of commodity taxes (e.g., import duties, sales taxes less subsidies), and international claims in the form of import commodity sales. After accounting for these claims, the net proceeds of commodity sales were claimed by the production activities that produced all commodities not imported.

[4] Lines connecting shapes in figure 1 that have arrows on both ends indicate both outbound and inbound financial flows.

Figure 1

Circular financial flow diagram for a social accounting matrix account



Source: USDA, Economic Research Service using information from the International Food Policy Research Institute.

Table 1

A simple three commodity example of a basic social accounting matrix (SAM)

		Production			**Institutional**			**International**	**Total**	
		Activity (*A*)	**Commodity (*C*)**	**Primary factors (*P*)**	**Consumption**		**Investment**			
		a1 *a2* *a3*	*c1* *c2* *c3*	*P*	*H*	*G*	*K*	*RoW* (Exports)		
Production	*a1* (Agriculture) *a2* (Manufacturing) *a3* (Other)		Domestic commodity sales by activity							Assemble commodities
	c1 (Farm commodities) *c2* (Goods) *c3* (Services)	Intermediate costs of production			Personal consumption	Government consumption	Capital accumulation and change in stocks	Export commodity sales		Demand for commodities
	P (Primary factors)	Gross value-added factor payments				Transfers to factor markets		Factor incomes from RoW		Factor income
Institutional	*H* (Household consumption)			Labor, mixed income, and distributed profits	Inter-household transfers	Transfers to households		Transfers to U.S. households		Institutional Receipts
	G (Government consumption)	Taxes less subsidies on output	Commodity taxes and duties	Labor, mixed income, and distributed profits	Direct taxes			Payments to U.S. Government		
	K (Investment)			Retained earnings	Personal savings	Government savings		Capital transfers from RoW		
International	*RoW* (Imports)		Imported commodity sales	Labor, mixed income, and distributed profits	Net transfers to RoW	Net transfers to RoW	International investment			Outlay to RoW
Total										
		Cost of production	Supply of commodities	Income distribution	Institutional outlays			Receipts from RoW		

Note: *RoW* = Rest of world.

Source: USDA, Economic Research Service.

The final subaccount of the production partition is the primary factor account. Proceeds to primary factors (*P*) include incomes from domestically owned primary factors employed by both domestic and international activities. For example, the proprietor who owns canning equipment to fill, seal, and label the tomato sauce has a claim on net proceeds from tomato sauce sales; however, had the proprietor instead leased this equipment those proceeds would have gone to lease payments. They also include all government transfers to factor markets (i.e., federal funding to invest in new technology). All primary factor market proceeds have multiple claimants, and these financial flows are depicted by the outward-bound arrows from the primary factor markets hexagon in figure 1. They include distributed primary factor earnings (i.e., labor, mixed income, distributed profits) to domestic institutions (i.e., households and government), and international claimants. In addition, some retained earnings are directed to investment for the accumulation of capital.

The consumption account consolidates all institutional spending by households and government for current consumption and directs this spending to commodity markets. This is described at the intersection of the commodity rows (*C*) and the household (*H*) and government (*G*) columns in table 1. Financial flows are depicted as passing through the consumption account from the institutional accounts and directed to commodity markets.

The investment account consolidated all savings, principally from domestic retained factor earnings, net household savings, and net capital inflows from international sources. It also facilitated account deficits such as from domestic governments, households, and the international account. All savings above that which covers current account deficits were invested through commodity purchases from domestic production, international imports, and direct international investments. This is all described in the investment (*K*) row and column of table 1, and the financial flows are depicted by inbound and outbound arrows from the "Investment" account depicted in figure 1.

The international account derives funds principally from import commodity sales in the United States with smaller amounts derived from claims on factor incomes from U.S. industries and transfers from Federal and State governments. Examples include distributions of dividends from foreign owned U.S. stocks, and U.S. Federal and/or State government dispersals through an international disaster assistance fund. Additional funds have flowed to this account from household remittances and U.S. investments abroad. Examples include private remittances of funds in both directions between U.S. households and their international relatives, and distribution of dividends from U.S. owned foreign stocks. These transactions are summarized across the international (*RoW*) row in table 1 and depicted by the inflow arrows to the international account in figure 1. International outflows to the United States are summarized down the international (*RoW*) column in table 1 and represent the reverse of the inflows to the international account. Any deficit in commodity trade in the international account was offset by an international surplus of transfers and investment with the United States.

2.1 Connecting Make and Use Tables to a Social Accounting Matrix Account

According to BEA, a make table shows the annual value, in producers' prices, of each commodity produced by each industry and a use table shows the value, in producers' prices, of each commodity used by each industry or by each final use and also shows the value added by each industry to produce its output. The connection between national make and use tables to a basic SAM account is depicted in figure 2 for a fictional national economy. The tables describe a fictional economy comprised of three industry groups (e.g., agriculture, manufacturing, other) and three commodity groups (i.e., farm commodities, goods, services).

Figure 2

A basic social accounting matrix account encompassing make and use tables

Figure 2a. Make table for fictional economy

Industries	Commodities: Farm commodities	Goods	Services	Total
	$-million			
Agriculture (box A→)	23	2		25
Manufacturing	4	128	3	135
Other			86	86
Total	27	130	89	

Figure 2b. Use table for fictional economy

Commodities/Industries	Intermediate uses: Agriculture	Manufacturing	Other	Final uses: PCE	PDI	Gov	Exp	Imp	Total commodity output
	$-million								
Farm commodities (box B→)	5	10	1	5	1	2	5	-2	27
Goods	5	40	25	50	15	5	10	-20	130
Services (box C)	5	30	25	50	4	1	5	-31	89
Gross operating surplus	10	55	35						
Total industry output	25	135	86	box D					box E

Figure 2c. Basic social accounting matrix (SAM) for fictional economy

	Production: Activity (*A*) *a1*	*a2*	*a3*	Commodity (*C*) *c1*	*c2*	*c3*	Factors (*F*) *F*	Institutional: Consumption *H*	*G*	Investment *K*	International *RoW* (Exports)	Total	
	$-million												
Production: *a1* (Agriculture)				23	2							25	Assemble commodities
a2 (Manufacturing)			box A→	4	128	3						135	
a3 (Other)						86						86	
c1 (Farm commodities)	5	10	1					5	2	1	5	29	Demand for commodities
c2 (Goods)	5	40	25	←box B				50	5	15	10	150	
c3 (Services)	5	30	25					50	1	4	5	120	
F (Factors)	10	55	35	←box C						box D		100	Factor income
Institutional: *H* (Household consumption)												0	Institutional receipts
G (Government consumption)												0	
K (Investment)												0	
International: *RoW* (Imports)			box -E'→	2	20	31						53	Outlay to *RoW*
Total	25	135	86	29	150	120	0	105	8	20	20		
	Cost of production			Supply of commodities			Income distribution	Institutional outlays			Receipts from *RoW*		

PCE = personal consumption expenditures. PDI = private direct investment. Gov = Government consumption and investment expenditures. Exp = exports. Imp = imports. *RoW* = rest of world.

Notes: Fictional economy indicates made up numbers not representing statistics from any actual country.

Source: USDA, Economic Research Service using made up numbers not representing statistics from any actual country.

Figure 2a reports the primary commodity outputs of each industry along the main diagonal of the interior table (excluding row and column totals) inside box A. All off diagonal data entries in this table report secondary outputs of each industry. The entire content of box A in the figure 2a make table can be copied into the intersection of activity rows and commodity columns of the target SAM account in box A of figure 2c. The entire content from three of four submatrices of the use table depicted in figure 2b (boxes B, C, & D) are also copied as is into the target SAM account in figure 2c (boxes B, C, & D). They include (box B) all intermediate uses of commodities by industries, copied as is into the intersection of *C* rows and *A* columns of the target SAM account in figure 2c; (box D) all final uses of commodities by institutional and international buyers, copied intact[5] into the intersection of *C* rows and the institutional plus international columns of the target SAM account in figure 2c; and (box C) all gross operating surplus by industries, copied as is into the intersection of *F* rows and *A* columns of the target SAM account. The remaining use table data in figure 2b is the international imports column that deducts commodity imports by recording as negative values. We simply pivot this import column, multiply values by -1, and copy into the intersection of the international (*RoW*) row and commodity (*C*) columns in figure 2c (box -E′). This is equivalent to reversing the flow of these transactions which requires a sign change.

Consider the target SAM account in figure 2c with all data from the national make and use tables systematically populated throughout the SAM account. Because the entire content of figures 2a and 2b (except totals and subtotals) is moved to the SAM, we are able to see that supply equals demand for each commodity and activity. We also find that the $100 million of gross domestic income (GDI) (i.e., total payments to primary factors by activities or the sum values in box C) plus the $53 million of total commodity imports from the international account (the sum values in box -E′) exactly equal the total dollars of final market commodity sales to institutional and international accounts (i.e., sum of values in box D). But we know that total institutional plus international final demand sales are equal to GDP plus imports so by extension we have GDI equaling GDP.

Although none of the rows *F*, *H*, *G*, *K*, or *RoW* in figure 2c show total receipts equal in value to total outlays reported in corresponding figure 2c's columns, we can collapse these rows to a single row measuring GDI plus imports, and collapse columns to a single column measuring GDP plus imports. The resulting table would be symmetric and balanced. Notice all the shaded areas in figure 2c, which correspond to transactions of a basic SAM account, described in these same sections of table 1 but not populated with any data were moved over from the make and use tables in figures 2a and 2b. If we narrow our focus to all shaded areas at the intersection of primary factor and institutional rows and columns (i.e., *F*, *H*, *G*, *K*), these shaded areas can be filled in through redistribution of outlays recorded from the source use table. For example, total factor incomes of 100 units reported across the primary factor row in figure 2c are recorded from the use table (figure 2b), yet no distribution of factor incomes are recorded down the primary factor (*F*) column in figure 2c. This information can be obtained from tables of the BEA NIPA accounts (USDOC, BEA, 2024f). For institutional rows in figure 2c (i.e., *H*, *G*, *K*), no receipts are recorded from the use table,[6] but commodity outlays across all three institutional columns are sourced from the use table. Institutional receipts by source can also be obtained from tables of the BEA NIPA accounts (USDOC, BEA, 2024f).

The only receipts and outlays of the international accounts obtained from the use table in figure 2b are the commodity imports and exports. Data for some of the shaded areas in the international column and row may be obtained from the BEA NIPA accounts and others can be imputed. For example, international claims on domestic factor incomes and domestic claims on international factor incomes are available

[5] The private direct investment (PDI) and government consumption and investment expenditures (Gov) data columns of the use table in figure 2b are flipped in order when copying data to figure 2c to follow the column order of the basic SAM.

[6] The practice of BEA use tables recording indirect output taxes on industry separate from gross operating surplus is not followed in figure 2b to simplify presentation.

(USDOC, BEA, 2024f). For other international transactions, such as private remittances between domestic and international households and international transfers with domestic government, NIPA accounts typically report net values (i.e., outflows minus inflows or vice versa). The ability to measure each component of net flows is greatly enhanced by the knowledge that the corresponding row and column totals in a balanced SAM account must be equal. For example, total outflows of U.S. dollars to the international account (i.e., row total) must be balanced by equal total inflows of U.S. dollars from the same account (i.e., column total). In the fictional economy depicted in figures 2a–2c, a commodity trade deficit of -33 (exports minus imports) must be offset by an equal trade surplus among all shaded international row and column transactions. Canning and Stacy (2019) has been an example for how BEA NIPA data, other Federal statistics, and SAM accounting identities can be combined with a reduced SAM (rSAM) account to compile a full and balanced SAM account.

2.2 A Note on the Scope and Boundaries of Ag-FEDS in this Report

We refer to an economic account that includes data on all the transactions described in table 1 as a basic SAM. A SAM that is compiled only from national make and use table data, as is depicted in figure 1, is missing all recorded transactions in the shaded areas of figure 2c and so is incomplete. We refer to this as a reduced SAM, or rSAM for short. It is sufficient to compile a subset of the structural model applications that are intended to be launched using Ag-FEDS from an rSAM of sufficient commodity and activity detail. Specifically, those applications that treat the primary factors (*F*), institutional (*H, G, K*), and international (*RoW*) accounts as exogenous[7] may be compiled from an rSAM of sufficient detail. Examples of such applications are a SAM multiplier adaptation of the Food Dollar multiplier model described in Canning (2011), and the resource multiplier application described in Canning et al. (2022).

An rSAM of sufficient activity and commodity detail also represents an ideal starting point to build out detailed SAM accounts of varying dimensions to address diverse areas of interest for USDA, ERS stakeholders. Previous USDA, ERS applications along these lines ranged from fixed price SAM multiplier models (Canning & Stacy, 2019) to applied general equilibrium structural simulation models at the national (Hanson & Hamrick, 2004) and multiregional (Canning & Tsigas, 2000) aggregation levels. Each of these applications take the rSAM in different directions in compiling more detailed SAM accounts. USDA, ERS has used these different applications to study a range of topics concerning fiscal policy impacts to the agri-food economy, USDA program impact analysis, and numerous other scenario analysis.

Given the practical and strategic benefits of having an rSAM of sufficient commodity and activity detail to support the range of applications, the scope of this report has been to develop such an rSAM as a timeseries to study both current and historical trends across the agri-food economy to ultimately produce the Agri-Food Economic Data System (Ag-FEDS). The approach will be to organize BEA annual and census year national make and use tables, various other supplemental BEA data products, and other official Federal Government statistics to compile a detailed system of accounts that we will assign to our target Ag-FEDS rSAM annual accounts spanning the years 1997 forward as new data become available. Further, we undertook a series of steps to organize our rSAM accounts to better capture the salient attributes of the U.S. agri-food economy that largely have been obscured by the initial rSAM based on BEA national make and use tables.

[7] Corresponding columns and rows of a SAM account are considered exogenous when their content remains unchanged during a structural simulation scenario from a model using the SAM account and may get updated after completion of the simulation.

Chapter 3. Development of the Agri-Food Economic Data System (Ag-FEDS)

The overarching goal in compiling the Ag-FEDS accounts is to provide a more complete and accurate accounting of food and beverage costs and resource use along the sequence of activities from farm production through points of purchase. This will involve three major components. First, we compiled a detailed and integrated system of annual reduced SAM accounts (**rSAM**) and personal consumption expenditure bridge (**PCEb**) tables based on and fully consistent with the integrated system of national accounts (SNA) and underlying detailed satellite accounts data published annually by the U.S. Department of Commerce, Bureau of Economic Analysis (USDOC, BEA, 2024e). This effectively doubled the number of activities and commodities explicitly accounted for in the current food dollar (Canning, 2011), resource use multiplier model (Canning et al., 2022), and SAM multiplier model (Canning & Stacy, 2019) applications. Second, we addressed issues of comingling among commodity flows along specific food value chains by reorganizing data and reconfiguring the **rSAM** accounts to disentangle misdirected commodity flows as measured using our multiplier model framework. Third, we measured the hidden food economy that is obscured in the detailed SNA data by integrating and reconciling key USDA, ERS data products published annually covering the U.S. agri-food economy. This effort accounted for between 4−11 percent more annual food expenditures within the food dollar framework, which allowed for a greatly expanded breakout of annual food commodity and marketing channel food dollar series and facilitated additional opportunities to disentangle farm commodities' flow.

An Explanation of Activities, Commodities, Price Levels, and Mathematical Notation

In this report, we present and discuss integrated accounts compiled using the social accounting matrix (SAM) accounting framework and notation convention. Activity-by-commodity accounting is more synonymous with SAM multiplier and computable general equilibrium (CGE) models, which will be the predominant research and data product application using the Agri-food economic data system (Ag-FEDS).

In SAM accounting it is conventional to refer to activities and commodities, described as follows:

- An activity is a grouping of establishments that produce one or more type of products often using a similar production process, instead of the input-output convention of referencing industries to describe the grouping of establishments producing only one product type. For example, tomato canning establishments may also grow their own tomatoes. Fresh tomatoes for canning and canned tomatoes are two products produced by this activity.
- A commodity results from the assembly of one type of product, acquired from one or more activities that produce this product, instead of the input-output convention where this assembly occurs before it is incorporated into the input-output account.

In national economic accounting, two price levels reported for many transactions are producer (*p*) prices and purchaser or market (*m*) prices:

- The producer price is the price received by the producer of a good or service before any shipping or handling (e.g., transportation, wholesale, and retail) charges are added.
- The purchaser or market price is the price paid by the buyer of a good or service after shipping and/or handling charges are added.

Mathematical notation in this chapter is as follows:

- A matrix is a rectangular array or table of numbers, arranged in rows and columns, and denoted with bold capitalized letters.
- A vector is a single column or row of numbers, denoted with bold lowercase letters.
- Sets are a predefined collection of elements inside of a matrix or vector and are denoted with capitalized and italicized letters.
- Set elements are specific individual or subgroup of elements within a set and are denoted with lower case italicized letters.
- A scalar is a single number and is denoted with nonbold lower case letters.
- Letters are from either the English or Greek alphabet.
- A matrix or vector transpose is denoted with a prime (′).
- A diagonal matrix has zeros off the main diagonal, at least one nonzero main diagonal element, and is denoted with a double prime (″).
- A matrix inversion is indicated by its placement inside brackets as $\{matrix\}^{-1}$
- A matrix to vector transformation by stacking columns from left to right is indicated by vec[*matrix*]
- Concatenations are either the vertical (//) or horizontal (||) joining of vectors and/or matrices to form a matrix having the combined number of rows (//) or columns (||) as the objects being joined.

Source: USDA, Economic Research Service adapted from Canning, P., Rehkamp, S., & Yi, J. (2022). Environmental input-output (EIO) models for food systems research: Application and extensions. In C.J. Peters & D.D. Thilmany (Eds.), Food systems modelling: Tools for assessing sustainability in food and agriculture (pp. 179-211). Elsevier.

3.1 Building Detailed Annual Accounts

In the SNA framework, a national make table typically reports the annual, market value of one or more commodities (*C*) produced by each industry group, here referred to as an activity (*A*) to be consistent with the **rSAM**, throughout a national economy. A national use table has three subaccounts. It reports the dollar value of intermediate outlays by activities on all the different commodities produced domestically and/or imported from international origins. The use table has also reported the dollar value of Gross Domestic Product (GDP) by commodity, measured as final market purchases of commodities by institutions (*I*) including governments (*g*), domestic households (*h*), capital/investor entities (*k*), and by a rest-of-world account (*R*), which measures international exports (*r1*) net of imports (*r2*) that BEA has recorded separately as a negative value, by commodity. The use table also has reported payments by all activities to primary factors of production (i.e., labor and capital inputs) and government output taxes and net fees[8] of government subsidies to the same group of institutions identified in the GDP accounts.

Consolidating national make and use tables within a unified data system has been a conventional practice when constructing **rSAM** tables. Some examples include Miller and Blair (2022), where a commodity-by-industry (*C*x*I*) table was compiled from supply or make (**S**)[9] and use (**U**) tables (for more information, see table 5.9 in Miller & Blair, 2022), and the United Nations (2018), where a commodity-by-industry table was compiled from **S**[10] and **U** tables (for more information, see box 2.1 in United Naitons, 2018). SAM multiplier model applications using this framework include Thorbecke (1998), Canning and Stacy (2019), and Canning et al. (2022).

The **rSAM** is an ideal framework to fully integrate the BEA **S**, **U**, and **PCEb** tables, both over time and across hierarchical account aggregation levels. BEA tier 0[11] aggregations of these accounts or their components include Detail (*D*), Underlying Summary (*Us*), Summary (*Su*), and Sector (*Se*). Tier 1 subset designations among the *D* accounts include *DA, DC, DI*, and *DR* representing the detailed production, consumption, institutional, and international accounts, respectively. Among *Su* accounts they are *SuA, SuC, SuI,* and *SuR*, and among *Us* accounts they are *UsA, UsC, UsI,* and *UsR*. With one minor exception involving margin tables, we did not work with any BEA *Se* aggregation tables. These tier 1 subaccounts all have one or more tier 2 sets that are fully contained within their boundaries. Any nonoverlapping set designations can be combined such that the symbol of these combined sets are the consolidated individual set symbols. For example, the combined activity/commodity set, $A \cup C$, is reported as *AC*. These BEA aggregations are hierarchical such that for all elements of the *D* aggregation, one or more elements map exactly into a single *Us* aggregation element, and for all elements of the *Us* aggregation, one or more elements map exactly into a single *Su* aggregation element. Table 2 lists all set, subset, and parameter designations used throughout this report.[12]

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[8] These do not include income and wage-based taxes that are levied by governments in the process of dispersing salaries and property incomes to primary factor owners; these taxes are not deducted from gross primary factor payments in a reduced SAM account.

[9] Some (e.g., Miller & Blair, 2022) interchangeably use **V** and **S** to denote the make table. Here we use '**S**'.

[10] The UN report works with supply tables in the same manner this report works with make tables. The information content in supply and make tables are very similar (for more information, see United Nations, 2018). However, the latter transposes data of the former such that activities are reported down rows and commodities across columns.

[11] In hierarchical accounting, tier 0 represents the highest-level hierarchy such that all account basic elements are members of tier 0. In the case of the SAM depicted in figure 2c, *rSAM* is the tier 0 account, and tier 1 members of this account are activities (*A*), commodities (*C*), primary factors (*F*), institutions (*I*), and international (*RoW*). An example of a tier 1 account is the set of all commodities (*C*) which itself could be further partitioned into tier 2 subgroups such as durable goods, nondurable goods, and various categories of services which can also have tier 3 subgroups.

[12] Throughout this report we use caloric as shorthand to refer to all food and beverage commodities purchased for human consumption. We recognize some commodities, such as bottled water, have no calories.

Table 2

Set, subset, and parameter designations

Tier 0 sets	Description	Tier 1 sets	Description	Tier 2 and [3] sets	Description
Se	BEA Sector level system of accounts	*SeA, SeC, SeI, SeR*	Subaccounts for Activities, Commodities, Institutions, Rest-of-world, Price levels (*O*)	*O (p, mg[tr, ws, rt], m)*	Price levels: producer price (*p*), total margin costs (*mg*)--comprising of transportation (*tr*) wholesale (*ws*) and retail (*rt*) trade--plus market prices (*m*).
Su	BEA Summary level system of accounts	*SuA, SuC, SuI, SuR, SuE, SuO*	Subaccounts for Activities (*A*), Commodities (*C*), Institutions (*I*), Rest-of-world (*R*), Expenditures (*E*), Price levels (*O*)	*I (h, g, k); R (r1, r2); O (p, m)*	Insitutional subaccounts: households (*h*), governments (*g*), and capital/financial (*k*). Rest-of-world: exports (*r1*) and imports (*r2*)
Us	BEA Summary Underlying Detail level system of accounts	*UsA, UsC, UsI, UsR*	Subaccounts for Activities, Commodities, Institutions, Rest-of-world		
D	BEA Detail level system of accounts	*DA, DC, DI, DR, DE, DO*	Subaccounts for Activities (*A*), Commodities (*C*), Institutions (*I*), Rest-of-world (*R*), Expenditures (*E*), Price levels (*O*)	*C (mg, xmg); I (h, g, k); R (r1, r2); O (p, mg, m)*	See table A.2 in Part IV of this report for tier 2 set descriptions from the *D* and *F* accounts
F	Ag-FEDS system of accounts	*FA, FC, FI, FR, FE, FO, FV, FVo*	Subaccounts for Activities (*A*), Commodities (*C*), Institutions (*I*), Rest-of-world (*R*), Expenditures (*E*), Price levels (*O*), vouchers (*V*), voucher overhead costs (*Vo*)	*A (mfg, nfs, fs, fsc, uv, xfs, fwfr, owor, mp, xmp, x045, SA, NA); C (mg, xmg, cen, xcen, nfs, fnb, fs, cal, fsc, fw, ow, fr, or, tr, fwfr, owor, veg, cala, calxa, farm, chem, mg2, ag, xag, SC, NC); I (h, g, k); R (r1, r2, mp); E (fbah, Xf, Xffah, Xfaah, XfFaw, Xfsm, Xflefa, XfFafh); O (p, m); V (psb, psp, psm, psf, psv, psom, psod, pro, SV), X (meat, xmeat, xmilk);*	
T ($\tilde{T}$)	Set of all benchmark year (nonbenchmark year) accounts	τ ($\tilde{\tau}$)	Benchmark (nonbenchmark) year account		

continued on next page ►

Table 2 (cont.)

Set, subset, and parameter designations

Parameter	Description	Parameter	Description
rSAM(1 to 4)	Intermediate iterations of target SAM account	**δQP**	Difference vector of linearized matrix vec(**QP1**-**QP0**)
vrSAM	Variance matrix for target SAM account	**vQP0**	Variance matrix for **QP0**
PCEb(1,2,3)	Intermediate iterations of target PCE bridge account	**fsshr**	Cost of all non-caloric inputs as a share of total sales by each type of commercial foodservice
vPCEb	Variance matrix for **PCEb** account	**Φ**	Mode shares for each freight service activity
S	National Supply (Make) matrix	**ω**	Wholesale (ws) needs share covered by food ws
U	National Use matrix	**deficit**	Shortfall of food ws available for food at home pce, and same for food retail
qU	Constant price Use matrix	**shrtrn**	Freight cost shares by mode for foodservices
Z	Endogenous transaction matrix	**shrFafh**	Share of FAFH deficit covered by FAH surplus
w	Unit price of endogenous transactions	**shrAafh**	Share of AAFH deficit covered by AAH surplus
Q	Constant price **Z** matrix	**XtraFrt**	Fresh fruit PCE difference to ERS measure
X (x)	Final demand matrix (vector)	**fsf**	Share of fresh fruit outlays among foodservices
L (l)	Primary factors matrix (vector)	**XtraVeg**	Fresh veggies PCE difference to ERS measure
v	GDI/imports multiplier vector	**rSAM**	Ag-FEDS Model dataset
y	Gross endogenous output vector	**QP0**	Prior matrix of quadraic program
qy	Constant price **y** vector	**QP1**	Posterior matrix of quadratic program
Ω	Row/column (dis)aggregation matrix	**Λ**	Direct requirement matrix
λ	Equation defined wildcard variable	**qΛ**	Constant price direct req. matrix
RPT	Number of nonzero summed values	**RAE**	Relative approximation error
pce	Market price PCE by *DE*	**scale**	Ratio of published PCE to prior detailed estimates

Note: Title case is used when referring to a specific *Xf* subaccount since these are our target tables (e.g., *Xf1101*).

Source: USDA, Economic Research Service.

The tier 0 aggregation for our target Ag-FEDS account is denoted *F* and corresponding tier 1 subaccounts are *FA, FC, FI,* and *FR*. *F* is a hybrid of the *D* and *Us* aggregations that enable USDA, ERS to fully leverage BEA annual data while also meeting the data product and research needs of USDA, ERS. Specifically, with the incorporation of all available detailed annual data resources at BEA, USDA, ERS, U.S. Department of Labor, Bureau of Labor Statistics (BLS), and the U.S. Department of Commerce, Bureau of the Census (Census Bureau), we can compile detailed structural models of annual food, beverage, and marketing channel supply chain market dynamics. With this detail we can do the following: (1) distinguish and assess the process of value formation over time and across food and beverage commodity groups and marketing channels to enhance our understanding of food price formation, food expenditure patterns, and supply-chain/market-structure dynamics as measured by the USDA, ERS Food Dollar data product; (2) more accurately link data on food choices and diet quality among the entire U.S. population and different subpopulations with detailed data and models of food and beverage production and international imports to study synergies and tradeoffs across diet, economic, and environmental outcomes; and (3) improve the economic accounting of supply chain linkages to better measure food and beverage commodity flows among and across different marketing channels so we can more accurately assess the macroeconomy implications of food assistance and food market policies. Figure 3 provides a schematic of our target **rSAM** accounting structure and table A.1 in the appendix provides a crosswalk between the Ag-FEDS *F* aggregation and the three BEA aggregations.

Figure 3 shows the organization of the **rSAM** table. It is organized for use with a SAM multiplier model having exogenous primary factor, institutional, and international accounts. For clarity, the schematic bypasses a primary factors account by directly distributing primary factor payments to institutional factor owners. This is an incomplete accounting of factor payment distributions, but sufficient for our purposes. In the top left block, the endogenous transactions matrix (**Z**) captures all annual bilateral transactions between activities and commodities and vice versa. Note that there are no activity-to-activity or commodity-to-commodity transactions, so quadrant 1 and 3 of the **Z** matrix are zeros,[13] where quadrants are numbered clockwise starting from the upper left quadrant. The injection matrix (**X**) shown in the top right block is the external account representing personal and government consumption and investment outlays for commodities, plus total international outlays for the export of domestic commodities. The leakage matrix (**L**) in the bottom left block represents outflows from the internal sectors going to primary factors and the rest of the world. Finally, the bottom right block is the **LX** matrix which represents exogenous transactions, or those outside of the model. Examples of these are direct transfer payments from the government account to households (e.g., social security benefits) that is not tied to current household employment or claims on other current period value added by industry. If we collapse the injection and leakage matrices depicted in figure 3 to column and row vectors respectively, all columns and rows depicted in figure 3 sum to either gross output (**y**) or GDP/GDI plus imports and each column sum (account dispersals) is equal to its corresponding row sum (account receipts). The data used to populate this target **rSAM** account were populated from BEA's make and use tables (figure 2). Our target *F* account aggregation is a combination of BEA's *D* level and *Us* level aggregations representing over 350 activities and commodities, 3 institutional accounts, and 2 international accounts (for a complete list with concordances to BEA aggregations, see table A.1 in the appendix).[14] In years when BEA published *D* level make and use tables (1997, 2002, 2007, 2012, 2017), we simply compiled our target **rSAM** account exactly as demonstrated in figure 2. In all other years, we compiled this account from numerous data sources. In all years we also established an integrated system of accounts that harmonized PCE bridge

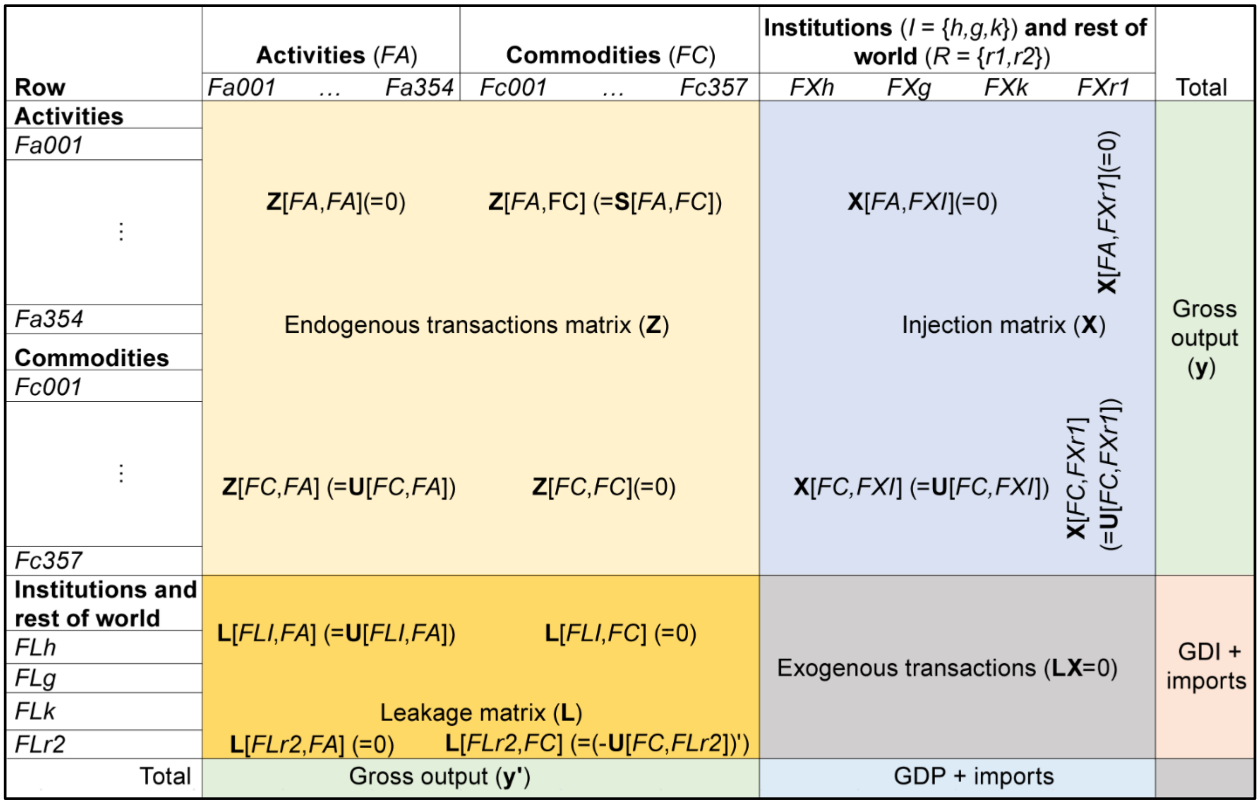
[13] Some sections of the schematic in figure 3 represent transactions that typically do not occur or are initially non-existent. Many, but not all these sections will later have recorded transactions. Here we depict all such sections by adding "=0" whether or not transactions will eventually be recorded in these sections.

[14] The exact number of activities and commodities depends on the source data release and step in the process of developing the **rSAM**.

accounts, and BEA's *Su* level make and use tables. To achieve this harmonization, we employed constrained maximum likelihood optimization models.

Figure 3

Reduced social accounting matrix (rSAM) schematic with exogenous institutional and international accounts



GDP = Gross Domestic Product. GDI = Gross domestic income. *FA* = activity subset of *F* account. *FC* = commodity subset of *F* account. *FXI* = Institutional subset of *F* account within injection matrix. *FLI* = Institutional subset of *F* account within leakage matrix. *FXr1* = Rest-of-world exports subset of *F* account within injection matrix. *FLr2* = Rest-of-world imports subset of *F* account within leakage matrix. **U** = use table.

Note: Institutional accounts (*I*) are partitioned into households (*h*), governments (*g*), and saving/investment (*k*). Rest-of-world accounts (*R*) are partitioned into international exports (*r1*), and international imports (*r2*).

Source: USDA, Economic Research Service adapted from Canning, P., Rehkamp, S., & Yi, J. (2022). Environmental input-output models for food systems research: Application and extensions. In C. J. Peters & D. D. Thilmany (Eds.), *Food systems modelling: Tools for assessing sustainability in food and agriculture* (pp. 179–211). Elsevier.

An Overview of the Constrained Maximum Likelihood Method

We want to verify the following: (1) that BEA benchmark year *Su* and *D* level make, use, and **PCEb** tables are fully integrated and can be compiled into a balanced **rSAM** at the target *F* aggregation level; and (2) that BEA's annual *Su* level make, use, and **PCEb** tables, BEA's *Us* level annual industry value added, and BEA's *D* level annual industry gross output and personal consumption expenditures at market prices (all published data products), plus other official government data products[15] can be compiled into balanced best general unbiased constrained annual estimates of **rSAM** accounts at the target *F* aggregation level.

[15] For example, Census Bureau monthly international trade statistics (U.S. Department of Commerce, Bureau of the Census, 2023b), and ERS Food Expenditure (USDA's Economic Research Service, 2024a) and various price spreads farm to consumer data products (USDA's Economic Research Service, 2024b & 2024c).

As discussed in Canning (2013), an early example of an efficient information processing approach to balancing economic data systems was the work of Stone et al. (1942), where a quadratic programming (QP) model was proposed for adjusting elements of the British national income accounts. Using relative reliabilities knowledge, we used a data updating procedure that weighs the penalties of adjusting initial estimates by their sample variance in the process of reconciling the integrated accounts. The theoretical underpinnings of the QP framework were based on a mathematical statistics foundation. For example, van der Ploeg (1988) demonstrated how a QP application with unbiased and normally distributed initial data can lead to best general unbiased constrained estimates. Weale (1985) demonstrated the maximum likelihood properties of the QP framework when initial estimates were distributed normally, which facilitates hypothesis testing. Harrigan (1990) demonstrated that even without imposing distributional assumptions on the data, constrained QP estimators were more efficient than the initial estimates, provided that the constraints introduced into the model are valid (validity is not an issue when working with data from SNA).

Methods of introducing information into the choice of initial estimates have received less emphasis in economics literature. When working with a structured data system such as an SNA, conditional estimators linked to primary and/or secondary data reduced the expected mean squared errors of initial estimates and can lead to uniformly minimum variance unbiased estimators of the unobserved target parameters, this is known as a Rao-Blackwellization (Rao, 1965). Canning and Wang (2005) demonstrated this property using a balanced international trade database with a known interregional trade matrix. The authors compared QP model solutions that alternatively used uninformed initial data via a data pooling procedure in one experiment and introduced a highly distorted version of the actual trade matrix in another experiment, and the latter uniformly produced estimates substantially closer to the actual trade matrix.

Several studies have examined hypothesis testing on QP estimators. Byron (1996) examined the performance of several standard tests for estimator bias using a constrained QP model to estimate a national SAM. The Monte Carlo simulations (1,000 replications) generated normally distributed SAM data priors with a bias sequentially introduced into otherwise unbiased random initial data. Byron (1996) found that a likelihood ratio test based on the ratio of the difference between constrained and initial estimates to the standard errors of data priors effectively identified initial estimate bias and "offered powerful support for the use of formal testing procedures." Preckel (2001) demonstrated that hypothesis testing under the QP framework was an interpretation of the general linear model. Yi et al. (2023) tested a constrained QP model of the type discussed here and used Monte Carlo and optimization techniques to recover suppressed data tables and used econometric models to evaluate the accuracy of imputations from alternative models. Various metrics of forecast accuracy show the flexibility and capacity of this approach to accurately recover suppressed data.

The remainder of this report will include nearly 150 equations reported in 8 subsections to facilitate replication and/or adaptation of the compiled Ag-FEDS product. To allow readers the option of following all the steps without having to read through every equation, each subsection that follows will be introduced with formatted summary covering objectives and steps caried out within the subsection, a brief description of every equation, and an orientation to the **rSAM** schematic in figures 3 and 5.

Benchmark Year Ag-FEDS Accounts

Compile benchmark year tier 0 accounts and subaccounts[16] and estimate Ag-FEDS as a constrained maximum likelihood (CML) problem:

- Equations 1–8 identify all BEA source data and their placements in the compiled *F* account.
- Equations 9–14 declare the initial target Ag-FEDS to BEA integrated data system and serve as constraints to the constrained maximum likelihood model specified in the next group of equations.
- Collectively, equations 1–14 replicate the steps depicted in figure 2 but for the actual U.S. economy using the **rSAM** account structure depicted in figure 3.
- Equations 15–16 define variables to measure changes from initial values of the endogenous variables for the CML model and converts matrices to vectors to facilitate specification of the maximum likelihood equation.
- Equations 17–18 constrain the endogenous variables to remain within a half unit (+/-) of their published priors.
- Equation 19 is the maximum likelihood objective function.
- Equations 15–19 refer to every element of the target **rSAM** accounts as depicted in figure 3, and **PCEb** accounts, for both the *F* and *Su* aggregations, and with constraint equations 9–14 ensure Ag-FEDS is fully integrated.

To compile regular and reliable data at the *F* level of detail we begin working with benchmark year accounts, $\tau \in T$, where $T = \{1997, 2002, 2007, 2012, 2017\}$. Denote $\mathbf{U}^{\tau p}$ the benchmark year τ use matrix with transactions reported in producer prices (for key term definitions and our math notation conventions, see box "An Explanation of Activities, Commodities, Price Levels, and Mathematical Notation"). Use matrices with a τm superscript report transactions recorded in market prices (producer and market prices are discussed below). Denote $\mathbf{S}^{\tau}$ the benchmark year τ supply or make matrix reporting domestic production of commodities by activities with values only reported in producer prices so no superscript is needed.

BEA publishes **S**, **U**, and **PCEb** U.S. tables at their *D* aggregation level only in benchmark years with a 5–6-year lag between source data enumeration and public release of the tables. For example, the most recent available benchmark enumeration year was 2017 with tables released starting in late 2023 (USDOC, BEA, 2023). In addition, BEA annually publishes **S**, **U**, and **PCEb** U.S. tables at their *Su* aggregation level with under a 1-year lag between source data enumeration and public release of the tables. For example, calendar year 2023 data tables were published by BEA in September 2024.

Given this cadence of BEA statistical releases, all data are available to populate an *F* aggregation of the complete **rSAM** table depicted in figure 3 only in a benchmark year (τ). All nonzero elements within the transaction (**Z**), injection (**X**), and leakage (**L**) matrices are populated with different sections from *F* aggregations of the *D* aggregation accounts, $\mathbf{S}^{\tau}$ and $\mathbf{U}^{\tau p}$.

Most transactions recorded in $\mathbf{rSAM}^{\tau}$ are measured in four segments, $O \in \{p, Setr, Sews, Sert\}$, representing producer prices[17] (*p*) plus the three margin (*Semg*) segments of transportation (*Setr*), wholesale (*Sews*), and retail (*Sert*) trade. The sector (*Se*) aggregation prefix for these margin account identifiers indicates that a single aggregated measure of each margin category is recorded. The combined

[16] When the entire row and/or column dimensions of target tier 0 accounts is included in an expression of that account or any of its subaccounts then only the tier 0 level reference is used (e.g., **rSAM**[*FACIr2*,*FACIr1*]=**rSAM**[*F*,*F*]).

[17] As an example, the producer price account depicted in figure 3 record all wholesale (retail) trade markups paid for purchases by activity *Fa001* of commodities *Fc001* to *Fc267* in the appropriate row among wholesale (retail) trade commodities *Fc269* to *Fc278* (*Fc279* to *Fc287*), and all transportation costs for these same commodity purchases are recorded in the appropriate row among transportation commodities *Fc288* to *Fc293*.

value of these four segments represents the market (m) value of each transaction, or purchaser value. In benchmark years only, BEA also reports use tables in all aggregation levels in purchaser prices, $\mathbf{U}^{\tau m}$, where all transactions are recorded in market prices and thus include the transportation, wholesale, and retail margin costs. We then deducted all these values from the margin commodity rows to derive rows that represent direct costs.[18] Also in benchmark years, all margin costs added to the purchaser prices of each transaction are reported ($\mathbf{U}^{\tau mg}$), and are further broken out into margin components, $\mathbf{U}^{\tau Semg} = \mathbf{U}^{\tau Setr} + \mathbf{U}^{\tau Sews} + \mathbf{U}^{\tau Sert}$. Information contained in $\mathbf{U}^{\tau Semg}$ and its components are important for the reorganization of the **rSAM** accounts, and so these tables must be part of the integrated system we intend to compile and carry forward to all nonbenchmark years. The identity among use table segments is stated as follows:[19]

$$\mathbf{U}^{\tau m} = \mathbf{U}^{\tau p} + \mathbf{U}^{\tau Semg} = \mathbf{U}^{\tau p} + \mathbf{U}^{\tau Setr} + \mathbf{U}^{\tau Sews} + \mathbf{U}^{\tau Sert}$$

This leaves an important gap in the information reported since we only know the level of transportation and trade margin costs of each transaction but do not know how much of each type of freight (*Ftr*) or trade service (*Fws, Frt*) is used. This gap must be recovered in the process of integration for our system of accounts.

To study food expenditures, we introduced tier 2 and tier 3 subaccounts. The *FI* subaccount is partitioned into three institutional tier 2 accounts (h, g, k) and the *FR* subaccount is partitioned into two international tier 2 accounts (*r1, r2*). *Fh* describes household personal consumption expenditures that are a tier 2 aggregation of several tier 3 expenditure categories (*hE*). The tier 3 disaggregation of PCE by expenditure category is reported at BEA by a separate **PCEb** table that disaggregates consumption expenditures reported in benchmark years into producer values plus the three margin segments ($\mathbf{PCEb}^{\tau}[\lambda C{\times}O,\lambda E]$) where $\lambda \in \{D,Su\}$,[20] broken out into 211 distinct detailed consumption expenditure categories (*DE*) and 76 distinct summary consumption expenditure categories (*SuE*), respectively. The information provided and relationship among segments, O, of the **PCEb** tables are the same as discussed above for the use tables. We integrated PCE bridge accounts into the system of accounts we want to carry forward to nonbenchmark years.

In order to accurately impute the gaps in our target F aggregation data system in nonbenchmark years, our target system of accounts must be fully integrated among all accounts (**rSAM**, **S**, **U**, **PCEb**) of a given aggregation and it must be fully integrated between all aggregation levels (F, D, Us, Su) and cost segments (p, $Setr$, $Sews$, $Sert$, m).

Before providing a mathematical statement of the fully integrated system, we explain our use of a versatile aggregation matrix. We designate $\mathbf{\Omega}$ for this matrix and denote its size with two subscripts defining row and column dimensions. For any matrix organized within a dimension hierarchy,[21] a matrix of any higher aggregation (parent) within that hierarchy is recovered from a lower aggregation matrix (child) using the row and column aggregation matrices as follows:

$$\mathbf{\Omega}_{\text{parent,child}} \times [\text{MATRIX}_{\text{child,child}}] \times \mathbf{\Omega}_{\text{child,parent}} = \text{MATRIX}_{\text{parent,parent}}$$

[18] For example, freight rail is a margin cost of a transaction whereas passenger rail is typically a direct expense.

[19] BEA does not report margin costs for margin commodity rows (*Fmg*), but these are recovered from the identity $\mathbf{U}^{\tau Semg}[Fmg,*] = \mathbf{U}^{\tau m}[Fmg,*] - \mathbf{U}^{\tau p}[Fmg,*]$ where * denotes all table columns, and this value (≤ 0) is assigned entirely to $\mathbf{U}^{\tau Setr}$, $\mathbf{U}^{\tau Sews}$, or $\mathbf{U}^{\tau Sert}$ when *mg* is a transportation, wholesale, or retail commodity respectively.

[20] Unlike the *Use* table, BEA does not consolidate margin outlays among *Fmg* rows in PCE bridge tables; rather, **PCEb** is recorded as a three-dimensional table, comprising of the Cartesian product of the commodity and cost-segment sets ($C{\times}O$), and expenditure category set (E).

[21] Except for the most aggregated dimension within this hierarchy.

A row or column aggregation matrix contains 1's in cells of all row-column combinations where the row (i.e., parent or child) element is related to the corresponding column (i.e., child or parent) element; otherwise, the cell contains a 0. The parent-child relationship for any aggregation matrix can be one or more generations apart, for example, *D* and *Su*.

A mathematical statement of the Ag-FEDS benchmark year integrated data system is as follows:

1.a $\mathbf{Z}^{\tau}[FAC,FAC] = ([0[FA,FA] \parallel \mathbf{\Omega}_{FA,DA} \times \mathbf{S}^{\tau}[DA,DC] \times \mathbf{\Omega}_{DC,FC}]) //$

1.b $([\mathbf{\Omega}_{FC,DC} \times \mathbf{U}^{\tau p}[DC,DA] \times \mathbf{\Omega}_{DA,FA} \parallel 0(FC,FC)])$

2. $\mathbf{X}^{\tau}[FAC,FIr1] = 0[\mathrm{F}A,FIr1] // (\mathbf{\Omega}_{FC,DC} \times \mathbf{U}^{\tau p}[DC,DIr1] \times \mathbf{\Omega}_{DIr1,FIr1})$

3.a $\mathbf{L}^{\tau}[FIr2,FAC] = [\mathbf{\Omega}_{FI,DI} \times \mathbf{U}^{\tau p}[DI,DA] \times \mathbf{\Omega}_{DA,FA} \parallel 0(FI,FC)] //$

3.b $[0[FA,Fr2] // \mathbf{\Omega}_{FC,DC} \times (-\mathbf{U}^{\tau p}(DC,Dr2) \times \mathbf{\Omega}_{Dr2,Fr2})]'$

4. $\mathbf{Z}^{\tau}[SuAC,SuAC] = (0[SuA,SuA] \parallel \mathbf{S}^{\tau}[\mathrm{Su}A,\mathrm{Su}C]) // (\mathbf{U}^{\tau p}[SuC,SuA] \parallel 0[SuC,SuC])$

5. $\mathbf{X}^{\tau}[SuAC,SuIr1] = 0[SuA,SuIr1] // \mathbf{U}^{\tau p}[SuC,SuIr1]$

6. $\mathbf{L}^{\tau}[SuIr2,SuAC] = (\mathbf{U}^{\tau p}[SuI,SuA] \parallel 0[SuI,SuC]) // (0[SuA,Sur2] // [-\mathbf{U}^{\tau p}(SuC,Sur2)])'$

7. $\mathbf{PCEb}^{\tau}[FC\times O,FE] = \mathbf{\Omega}_{FC\times O,DC\times O} \times \mathbf{PCEb}^{\tau}[DC\times O,DE] \times \mathbf{\Omega}_{DE,FE} + \mathbf{IP1}^{\tau}[FC\times O,FE]$

8.a $\mathbf{rSAM}^{\tau}[\lambda,\lambda] = (\mathbf{Z}^{\tau}[\lambda AC,\lambda AC] \parallel \mathbf{X}^{\tau}[\lambda AC,\lambda Ir1]) // (\mathbf{L}^{\tau}[\lambda Ir2,\lambda AC] \parallel 0[\lambda Ir2,\lambda Ir1])$

8.b $+ \mathbf{IP2}^{\tau}[\lambda,\lambda], \forall \lambda \in \{F,Su\}$

Declare the initial Ag-FEDS to BEA integrated data system ($\mathbf{rSAM1}^{\tau}$ and $\mathbf{PCEb1}^{\tau}$):

9. $(\mathbf{\Omega}_{1,\lambda} \times \mathbf{rSAM1}^{\tau}[\lambda,\lambda AC])' = \mathbf{rSAM1}^{\tau}[\lambda AC,\lambda] \times \mathbf{\Omega}_{\lambda,1}, \forall \lambda \in \{F,Su\}$

10. $\mathbf{\Omega}_{1,\lambda Ir2} \times \mathbf{rSAM1}^{\tau}[\lambda Ir2,\lambda AC] \times \mathbf{\Omega}_{\lambda AC,1} = \mathbf{\Omega}_{1,\lambda AC} \times \mathbf{rSAM1}^{\tau}[\lambda AC,\lambda Ir1] \times \mathbf{\Omega}_{\lambda Ir1,1}, \forall \lambda \in \{F,Su\}$

11. $\mathbf{\Omega}_{Su,F} \times \mathbf{rSAM1}^{\tau}[F,F] \times \mathbf{\Omega}_{F,Su} = \mathbf{rSAM1}^{\tau}[Su,Su]$

12. $\mathbf{\Omega}_{SuC\times O,FC\times O} \times \mathbf{PCEb1}^{\tau}[FC\times O,FE] \times \mathbf{\Omega}_{FE,SuE} = \mathbf{PCEb1}^{\tau}[SuC\times O,SuE]$

13. $\mathbf{rSAM1}^{\tau}[Fxmg,Fh] = \mathbf{PCEb1}^{\tau}[Fxmg\times p,FE] \times \mathbf{\Omega}_{FE,1}$

14.a $\mathbf{\Omega}_{Semg,Fmg} \times \mathbf{rSAM1}^{\tau}[Fmg,Fh] = \mathbf{\Omega}_{Semg,Fmg\times p} \times \mathbf{PCEb1}^{\tau}[Fmg\times p,FE] \times \mathbf{\Omega}_{FE,Fh} +$

14.b $\mathbf{\Omega}_{Semg,FC\times Semg} \times \mathbf{PCEb1}^{\tau}[FC\times Semg,FE] \times \mathbf{\Omega}_{FE,Fh}$

Equations 1–8 identify all BEA source data and their precise placement in the compiled tier 0 *F* accounts. Equations 1–3 demonstrate that the target benchmark year $\mathbf{rSAM}^{\tau}$ *F* account is fully determined from components of the published BEA make ($\mathbf{S}^{\tau}$) and use ($\mathbf{U}^{\tau p}$) tables. The entire make table and the intermediate activity transactions subaccount of the use table are used to fully populate the target endogenous transactions subaccount ($\mathbf{Z}^{\tau}$). Remaining data from the use table are systematically partitioned to populate the injection ($\mathbf{X}^{\tau}$) and leakage matrices ($\mathbf{L}^{\tau}$). Equations 4–6 demonstrate the same set of relationships between the more aggregated summary level make and use tables. Equation 7 shows that the target benchmark year $\mathbf{PCEb}^{\tau}$ account is fully determined from aggregation of the same account published by BEA for benchmark years at the *D* aggregation level. When $\lambda = F$ equation 8 is the mathematical equivalent of figure 3 and states the precise structure of our target Ag-FEDS **rSAM** account for any benchmark year. When $\lambda = Su$, equation 8 compiles the same account published by BEA for benchmark years and annually at the *Su* aggregation level.

Equations 7 and 8 include ill-posed (IP) matrices **IP1** and **IP2,** respectively, to cover scenarios (more common in nonbenchmark years), where a *Su* level transaction in either tier 0 account has no prior information of transactions in their corresponding *F* level account. Because these cases are unusual, we describe their estimation rather than introduce new notation. Estimation involves allocating the *Su* level value equally to all candidate *F* level transactions that map into this *Su* level transaction. For example, a nonbenchmark year *S* level make table that shows a small amount of scrap as a byproduct of an activity that produced no scrap in any year since the most recent benchmark year *D* level account was published. Because no prior information existed to inform estimates of scrap byproducts from any *Su* level activity in the non-benchmark year, the mean and variance moment priors for these data cells are 0, which creates an infinite penalty (division by 0 in the likelihood equation below) to adjust the 0 prior value. This produces a violation of equation 11 such that the problem becomes ill posed. The simple work around in **IP1** is to assign equal prior probabilities to all candidate values of *F* level data cells in **rSAM**[*F*,*F*] having no prior information, unless specific valid information exists that informs a different distribution. In both a theoretical statistics and Bayesian context an equal probability prior is one that offers no additional information about the parameter being estimated. Examples of this involve small values, and the equal probability prior is consistent with the likelihood equation framework because the prior estimates described above for parameters **IP1** and **IP2** are unbiased (Byron, 1996).

Equations 9–14 declare the initial target Ag-FEDS to BEA integrated data system. Equations 9 and 10 are the necessary accounting properties of any well posed SAM account. Equation 9 states the sum of annual domestic production plus international imports of each activity and commodity (supply) exactly equals the sum of annual domestic acquisitions plus international exports of each activity and commodity (use). Equation 10 states gross domestic income (GDI) measured as total payments to institutions (e.g., household labor, property owners, and government) by all domestic activities exactly equals Gross Domestic Product (GDP) measured as final domestic market commodity sales plus net exports (exports minus imports). Both sides of the equality in equation 10 include the total value of international commodity imports such that deducting this total from both sides make up GDI and GDP. When $\lambda = Su$, the interpretation of equations 9 and 10 is the same. However, the accounts are aggregated into far fewer activities and commodities.

A precise relationship between the *Su* and *F* level **rSAM** accounts is established by equation 11. This is important since, unlike the *D* level BEA accounts that fully populate our target **rSAM** matrix in benchmark years only, all the data to populate the *Su* level **rSAM** are published annually with a less than a 1-year lag from statistical year and public data release. The same holds between the *Su* and *F* level **PCEb** accounts as established by equation 12, and these *Su* level accounts are also published annually. The relationships described in equations 11 and 12 serve to inform what will be our underdetermined nonbenchmark year data system described in the next section.

To close the system of integrated accounts, the precise relationship between our target tier 0 accounts is described in equations 13 and 14. Five transportation freight modes, 10 wholesale, and 9 retail trade services help facilitate transactions of goods throughout the economy and costs added to transactions for these services are called margins, so these 24 commodities are called margin commodities (*Fmg*) and all other commodities comprise the group nonmargin commodities (*Fxmg*). Personal consumption expenditures on nonmargin commodities, denoted *Fxmg*, are fully described in both tier 0 accounts, and equation 13 compiles these data and establishes their equality in both accounts. Among the 24 margin commodities, it is important to note that personal consumption expenditures on some of these commodities reflect consumption expenditures, such as airline tickets for personal travel, but most expenditures on these commodities are for margin costs from acquisition of other goods. In the **rSAM** account, total personal consumption expenditures on all 24 margin commodities were recorded without distinction as a consumption or margin cost. In the **PCEb** account only consumption expenditures on margin commodities were recorded, whereas total margin costs were recorded at the BEA sector level (*Se*), which aggregated the 24 *F* level commodities into one commodity each for transportation, wholesale, and retail. For this reason, a precise relationship between the two tier 0 accounts for margin commodities required both accounts be aggregated to the sector level. For the **rSAM** account, this is described in the left side of the equality in equation 14.a. For the **PCEb** account, the consumption costs aggregation to the *Se* level is described on the right side of the equality in equation 14.a, whereas the margin cost aggregation is described in equation 14.b and both lines together establish the *Se* level equality of margin commodity expenditures in both accounts.

In benchmark years, our target Ag-FEDS account defined by equations 1–14 is well-posed and fully determined by published and publicly available data (USDOC, BEA, Industry Economic Accounts, 2023). However, because published source data tables were reported as integers in million dollar units the stated equalities in many equations are violated due to rounding errors. This can be routinely addressed, and in doing so we established the following: (1) BEA published make, use, PCE bridge, and margin data tables that were fully integrated and free of specification errors, which has been important when confronting those rare cases where source BEA data were published in error; and (2) a relaxation of the integer data constraint facilitated specification of the constrained maximum likelihood (CML) equation, which ensured that, among the many possible well-posed solutions, we found the best linear unbiased efficient estimates (Weale, 1985). Our Ag-FEDS account was estimated as a constrained maximum likelihood problem ($\mathscr{L}$) as follows:

15. $\boldsymbol{\delta rSAM}^{\tau}[\lambda,\lambda] = (\text{vec}[\ \mathbf{rSAM1}^{\tau}[\lambda,\lambda] - \mathbf{rSAM}^{\tau}[\lambda,\lambda]\ ])''$

16. $\boldsymbol{\delta PCEb}^{\tau}[\lambda,\lambda] = (\text{vec}[\mathbf{PCEb1}^{\tau}[\lambda,\lambda] - \mathbf{PCEb}^{\tau}[\lambda,\lambda]])''$

17. $(\mathbf{rSAM}^{\tau}[\lambda,\lambda] - 0.5\times\mathbf{RPT}[\lambda,\lambda]) < \mathbf{rSAM1}^{\tau}[\lambda,\lambda] < (\mathbf{rSAM}^{\tau}[\lambda,\lambda] + 0.5\times\mathbf{RPT}[\lambda,\lambda])$

18. $(\mathbf{PCEb}^{\tau}[\lambda,\lambda] - 0.5\times\mathbf{RPT}[\lambda,\lambda]) < \mathbf{PCEb1}^{\tau}[\lambda,\lambda] < (\mathbf{PCEb}^{\tau}[\lambda,\lambda] + 0.5\times\mathbf{RPT}[\lambda,\lambda])$

19.a $\text{Max}[\mathscr{L}l(\mathbf{rSAM1}^{\tau},\mathbf{PCEb1}^{\tau})] = -0.5\times\Omega_{\lambda}[\ \boldsymbol{\delta rSAM}^{\tau}[\lambda,\lambda]\times\{\text{vec}[\mathbf{vrSAM}^{\tau}[\lambda,\lambda]]\}^{-1}\times\boldsymbol{\delta rSAM}^{\tau}[\lambda,\lambda] +$

19.b $\boldsymbol{\delta PCEb}^{\tau}[\lambda,\lambda]\times\{\text{vec}[\mathbf{vPCEb}^{\tau}[\lambda,\lambda]]\}^{-1}\times\boldsymbol{\delta PCEb}^{\tau}[\lambda,\lambda]\ ],\ \forall\ \lambda\in\{F,Su\}$

$\mathbf{rSAM1}^{\tau}[\lambda,\lambda]$ are the endogenous variables representing the unrounded $\mathbf{rSAM}^{\tau}[\lambda,\lambda]$ matrix, and $\mathbf{vrSAM}^{\tau}[\lambda,\lambda]$ is the variance matrix measuring the unreported variance of elements in the $\mathbf{rSAM}^{\tau}[\lambda,\lambda]$ accounts $\forall\ \lambda\in\{F,Su\}$. $\mathbf{PCEb1}^{\tau}[\lambda,\lambda]$ are the endogenous variables representing the unrounded $\mathbf{PCEb}^{\tau}[\lambda,\lambda]$ matrix, and $\mathbf{vPCEb}^{\tau}[\lambda,\lambda]$ is the variance matrix measuring the unreported variance of the $\mathbf{PCEb}^{\tau}[\lambda,\lambda]$ accounts $\forall\ \lambda\in\{F,Su\}$. If we assume the unreported coefficients of variation among $\mathbf{rSAM}^{\tau}$ and $\mathbf{PCEb}^{\tau}$ elements are the same (all published values are equally reliable), we can use the reported $\mathbf{rSAM}^{\tau}$ and $\mathbf{PCEb}^{\tau}$ as linear unbiased transformations of their variance matrices, and equation 19 is the likelihood equation (Weale, 1985; Byron, 1996; Canning, 2013; Yi et al., 2023). Maximization of 19 subject to constraint equations 9–18 represents the CML solution to our target benchmark year Ag-FEDS account. This CML model routinely solved in all benchmark years as expected since it is simply a well-posed aggregation of the published integrated BEA SNA. Equations 17 and 18 ensured that the unrounded solution exactly recovers the published accounts when reported as integers. For $\lambda = F$ in equations 17 and 18, the matrix **RPT** reports the number of nonzero integers in the source *D* level matrix that are added to create an *F* level value, since each source integer has the potential to have been rounded up or down.[22] However, BEA data tables report blank data cells if there is no measured value and report a 0 in data cells that round to 0. We changed these published zeros to 0.25 and add 0.5 to the corresponding element in **RPT** for each such source data cell. For $\lambda = Su$ all elements equal 1, 0.5, or 0 in the **RPT** since there is no

[22] By a value < 0.5.

aggregation of source data. For $\lambda = F$ all elements are multiples 1, 0.5, or 0 in the **RPT** since there may be aggregation of source data.[23]

We also carried forward the margin accounts, $\mathbf{rSAM}^{\tau\lambda}[FC,FAI]$ for $\lambda \in$ (*Setr*, *Sews*, *Sert, m*). These accounts are compiled from published *D* level BEA benchmark year tables (USDOC, BEA, 2024b). The decimal precision was not needed for the margin accounts because we only carried forward the sector level (*Se*) detail of these margin values, which covers one commodity each for all transportation (*Setr*), wholesale (*Sews*) and retail (*Sert*) commodity.

Nonbenchmark Year Ag-FEDS Accounts

Compile annual tier 0 accounts and subaccounts. Revised statements of equations 1−3, 7, 17, and 18 complete the initial nonbenchmark year Ag-FEDS integrated system initial estimates:

- Equation 20 specifies the direct requirement matrix (**Λ**) for activities and commodities within the **Z** matrix and demonstrates that nominal values in **Z** and **y** are products of volume, denoted **qZ** and **qy**, and unit prices, denoted **w.**
- Equation 21 demonstrates derivation of the benchmark year constant price direct requirement matrix (**qΛ**), a section of which (**qΛ**[*FC,FA*]) is hypothesized to be time-invariant between benchmark years.
- Equation 22 derives the price index vector as a hybrid approach relying on published BEA price data for activities and weighted average commodity prices based on current information for activity composition of commodities.
- Equations 20 to 22 are input parameters to implement our approach to developing prior estimates for all elements of our target nonbenchmark year **rSAM** and **PCEb** accounts, to serve as inputs to our maximum likelihood estimates:
- To recover the unpublished nonbenchmark year *F* level make and use tables to populate our target **rSAM** and **PCEb** annual accounts, the equations below leverage our null hypothesis on the various time-invariant, unit volume requirement parameters and the following published data:
    - BEA Industry Economic Accounts data on (1) annual *D*-level gross activity output (real and nominal), (2) annual *Us*-level activity value added, and (3) annual *Su*-level supply, use, and **PCEb** tables.
    - BEA National Income and Product (NIPA) *D*-level personal consumption expenditures data by expenditure category in market prices.
    - Census Bureau annual detailed North American Industrial Classification System (NAICS) data on (1) commodity imports and exports, and (2) activity value added.
- Equation $2^{\sim}$ introduces **qX,** which is analogous to **qZ** (from equation 20) and measures constant price (volume) units for the injection submatrix (figure 3) and employs the time invariant hypothesis to its benchmark year values for imputations in $2^{\sim}$.
- A **scale** parameter used in equation $7^{\sim}$ and defined in equation $18^{\sim}$ ensures all initial and maximum likelihood estimates for **PCEb** replicate the published NIPA PCE market prices table values.

[23] For example, if three elements of the source BEA *D* level *Use* table aggregate to a cell in the target *F* level **Z** matrix and these source elements are reported as one nonzero integer, one 0, and one blank cell, the corresponding **RPT** matrix element would be 1.5, such that equation 17 would allow the initial **Z** cell element prior to vary by less than +/- 0.75 (0.5×1.5).

- Equations $1^{\sim}$–$3^{\sim}$, $7^{\sim}$, $17^{\sim}$, and $18^{\sim}$ combined with restatement of equations 4–6 and 8–14 plus equation 19 replacing τ with $\tau^{\sim}$ represents the complete constrained maximum likelihood model for nonbenchmark years and produces the full annual **rSAM** and **PCEb** Ag-FEDS accounts.
- Equation 23 defines the relative approximation error statistic that is applied to every estimated value in all benchmark and annual Ag-FEDS accounts and form the basis for statistical evaluation of estimated results reported in figure 4.

For any year $\tau^{\sim}$, where $\tau^{\sim} > \tau$ and $\tau^{\sim} \notin T$, an incomplete set of BEA D statistics are reported and so our target Ag-FEDS account is not fully determined. We seek to compile an unbiased prior account $\mathbf{rSAM}^{\tau^{\sim}}$ and populate with mean and variance data derived from year $\tau^{\sim}$ primary statistical sources.

To facilitate this objective, we introduce the following direct requirement technology matrix:

20.a $\mathbf{\Lambda}^{\tau(\sim)}[FAC,FAC] = \mathbf{Z}^{\tau(\sim)}[FAC,FAC] \times \{\mathbf{y}^{\tau(\sim)}[FAC]''\}^{-1}$

20.b $= ((\mathbf{w}^{\tau(\sim)}[FAC])'' \times \mathbf{qZ}^{\tau}[FAC,FAC]) \times \{(\mathbf{w}^{\tau(\sim)}[FAC])'' \times \mathbf{qy}^{\tau(\sim)}[FAC])''\}^{-1}$

$\mathbf{\Lambda}$ is the direct requirement matrix, measuring total nominal dollars of each commodity $c \in C$ required per nominal dollar of output for each activity $a \in A$. It also measures total nominal dollars of each activity $a \in A$ required per nominal dollar of output plus imports for each commodity $c \in C$. Recall that vector **y** measures gross domestic activity outputs and total domestic commodity outputs plus commodity imports (figure 3). Units are measured in current year prices, $\mathbf{w}^{\tau(\sim)}$, such that direct requirement coefficients will vary over time as relative prices vary, even when the underlying technologies remain unchanged. The **q** prefix denotes volume which is the constant price analog to the matrix or vector it precedes.

In contrast, a constant price direct requirement matrix will remain constant over time when underlying technologies remain unchanged. They can be calibrated with full information in benchmark years and are measured as follows:

21.a $\mathbf{q\Lambda}^{\tau}[FAC, FAC] = \{\mathbf{w}^{\tau}[FAC]''\}^{-1} \times \mathbf{Z}^{\tau}[FAC, FAC] \times \{\{\mathbf{w}^{\tau}[FAC]''\}^{-1} \times \mathbf{y}^{\tau}[FAC]''\}^{-1}$

21.b $= \mathbf{qZ}^{\tau}[FAC,FAC] \times \{(\mathbf{qy}^{\tau}[FAC])''\}^{-1}$

In calibrating our Ag-FEDS account in nonbenchmark years, our null hypothesis is $\mathbf{q\Lambda}[FC, FA]$ remains constant between benchmark years. Our approach for $\mathbf{q\Lambda}[FA, FC]$ differs due to data availability in nonbenchmark years. If for $\mathbf{qZ}^{\tau}[FA,FC]$ we were to normalize each element by its row total (gross activity output), we denote this parameter $\mathbf{q'\Lambda}$ which should not be confused with the transpose of $\mathbf{q\Lambda}$. Our null hypothesis for submatrix $\mathbf{q'\Lambda}[FA, FC]$ is that the previous year values for this submatrix are best linear unbiased estimates of current year values.

The annual BEA underlying detail Industry Economic Accounts (IEA) and National Income and Product Accounts (NIPA), plus several annual Census Bureau data products are available to develop an initial ill-posed target $\mathbf{rSAM}^{\tau^{\sim}}$ account. From BEA, these data products include annual D level nominal gross activity output ($\mathbf{y}^{\tau^{\sim}}[DA]$) and real (constant price) gross activity output ($\mathbf{qy}^{\tau^{\sim}}[DA]$), from which we can derive a unit price index ($\mathbf{w}^{\tau^{\sim}}[DA] = \{\mathbf{qy}^{\tau^{\sim}}[DA]''\}^{-1} \times \mathbf{y}^{\tau^{\sim}}[DA]$). Also from BEA are the Us level nominal gross primary factor value added by activity ($\mathbf{U}^{\tau^{\sim}}[UsI,UsA]$) and real gross primary factor value added by activity ($\mathbf{qU}^{\tau^{\sim}}[UsI,UsA]$), from which we can derive a unit primary factor price index ($\mathbf{v}^{\tau^{\sim}}[UsA] = \{\mathbf{qU}^{\tau^{\sim}}[UsI,UsA]''\}^{-1} \times \mathbf{U}^{\tau^{\sim}}[UsI,UsA]'$). Gross output and value-added data come from the IEA underlying detailed annual accounts. A D-level annual personal consumption expenditure table reported in market prices ($\mathbf{pce}^{\tau^{\sim}}(DE)$)[24] is available from BEA's NIPA underlying detailed annual accounts. Also from BEA, all the relevant Su level data identified in equations 4–12 are published annually.

[24] Unlike the **PCEb** table of the IEA acconts, NIPA **pce** tables do not map expenditures (E) to commodities (C).

Additional annual data that we used to inform our initial target account come from the Census Bureau. All the Census Bureau statistics are NAICS[25] (N) level data that can be exactly aggregated to F-level manufacturing activities and all F-level traded commodities, respectively. These data include all manufacturing activity (mfg) gross value-added data ($\mathbf{U0}^{\tilde{\tau}}[FLI,NAmfg]$)[26] from their Annual Survey of Manufacturing (USDOC, Bureau of the Census (Census Bureau), 2023a), and annual international trade data (USDOC, Census Bureau, 2023b) for both total international commodity imports ($-\mathbf{U0}^{\tilde{\tau}}[NC,Fr2]$) and exports ($\mathbf{U0}^{\tilde{\tau}}[NC,Fr1]$). The annual unit commodity price indexes ($\mathbf{w}^{\tilde{\tau}}[DC]$), and a method to derive F level commodity prices are missing from these sources. We can compile the F-level activity price indexes directly from the published annual nominal and real gross output data from BEA. However, in this accounting framework commodity price indexes are weighted averages of activity prices, and the current year activity composition of commodities is not theoretically known. Recalling our null hypothesis concerning the activity by commodity submatrix within the $\mathbf{q\Lambda}$ account, our approach was to weight current year activity composition of commodities by the previous year's maximum likelihood or published benchmark year weights:[27]

22.a $\mathbf{w}^{\tilde{\tau}}[FAC] = \{\mathbf{qy}^{\tilde{\tau}}[FA]''\}^{-1} \times \mathbf{y}^{\tilde{\tau}}[FA]$

22.b $\quad // \; (\mathbf{qZ}^{(\tilde{\tau}-1)}[FA,FC] \times \{(\mathbf{\Omega}_{1,FA} \times \mathbf{qZ}^{(\tilde{\tau}-1)}[FA,FC])''\}^{-1})' \times (\{\mathbf{qy}^{\tilde{\tau}}[FA]''\}^{-1} \times \mathbf{y}^{\tilde{\tau}}[FA])$

where the transposed previous year activity composition share matrix in 22.b translates F-level activity prices into weighted average commodity prices. We used row shares for the FC subsection of vector $\mathbf{w}^{\tilde{\tau}}[FAC]$ because of the availability of detailed current year activity prices.

We worked to recover the unpublished nonbenchmark year F-level make and use tables, adopting the accounting methods used by BEA (for more information, see chapter 12 in BEA (2018), and chapter 5 in Miller & Blair (2022)). With these data, the initial ill-posed nonbenchmark year Ag-FEDS integrated system is a restatement of equations 4–6, equations 8–14 replacing τ with $\tilde{\tau}$, and the following restatement of other benchmark year equations:[28]

$\tilde{1}$.a $\mathbf{Z}^{\tilde{\tau}}[FAC,FAC] = (0[FA,FA] \,\|\, \mathbf{y}^{\tilde{\tau}}[FA]'' \times [\{(\mathbf{q'\Lambda}^{\tau-1}[FA,FC] \times \mathbf{w}^{\tilde{\tau}}[FC])''\}^{-1} \times$

$\tilde{1}$.b $\quad (\mathbf{q'\Lambda}^{\tau-1}[FA,FC] \times \mathbf{w}^{\tilde{\tau}}[FC]'')]) \; // \; (\mathbf{w}^{\tilde{\tau}}[FC]'' \times \mathbf{q\Lambda}^{\tau}[FC,FA] \times$

$\tilde{1}$.c $\quad \{(\mathbf{w}^{\tilde{\tau}}[FC]' \times \mathbf{q\Lambda}^{\tau}[FC,FA])''\}^{-1} \times (\mathbf{y}^{\tilde{\tau}}[FA] - (\mathbf{\Omega}_{1,FI} \times \mathbf{L}^{\tilde{\tau}}[FI,FA])') \,\|\, 0[FC,FC])$

$\tilde{2}$.a $\mathbf{X}^{\tilde{\tau}}[FAC,FIr1] = 0[FA,FIr1] \; //$

$\tilde{2}$.b $[(\mathbf{\Omega}_{Fxmg,FC\times p} \times \mathbf{\Omega}_{FC\times p,FC\times O} \times \mathbf{PCEb}^{\tilde{\tau}}[FC\times O,FE] \times \mathbf{\Omega}_{FE,1} \; // \; \mathbf{U}^{\tilde{\tau}}[Sutr,Sh] \; //$

$\tilde{2}$.c $[\{(\mathbf{w}^{\tilde{\tau}}[Fws]' \times \mathbf{qX}^{\tau}[Fws,Fh])''\}^{-1} \times (\mathbf{w}^{\tilde{\tau}}[Fws]'' \times \mathbf{qX}^{\tau}[Fws,Fh])] \times \mathbf{U}^{\tilde{\tau}}[Suws,Suh] \; //$

$\tilde{2}$.d $[\{(\mathbf{\Omega}_{Frt,Surt} \times (\mathbf{\Omega}_{Surt,Frt} \times \mathbf{w}^{\tilde{\tau}}[Frt]'' \times \mathbf{qX}^{\tau}[Frt,Fh]))''\}^{-1} \times (\mathbf{w}^{\tilde{\tau}}[Frt]'' \times \mathbf{qX}^{\tau}[Frt,Fh])]'' \times \mathbf{\Omega}_{Frt,Surt} \times \mathbf{U}^{\tilde{\tau}}[Surt,Suh]) \,\|$

$\tilde{2}$.e $(\mathbf{X}^{\tau}[FC,Fk]'' \times \mathbf{\Omega}_{FC,SuC} \times \{\mathbf{U}^{\tau}[SuC,Suk]''\}^{-1} \times \mathbf{\Omega}_{FC,SuC} \times \mathbf{U}^{\tilde{\tau}}[SuC,Suk]) \,\|$

$\tilde{2}$.f $([\{(\mathbf{\Omega}_{FC,SuC} \times (\mathbf{\Omega}_{SuC,FC} \times \mathbf{w}^{\tilde{\tau}}[FC]'' \times \mathbf{qX}^{\tau}[FC,Fg]))''\}^{-1} \times (\mathbf{w}^{\tilde{\tau}}[FC]'' \times \mathbf{qX}^{\tau}[FC,Fg])]'' \times \mathbf{\Omega}_{FC,SuC} \times \mathbf{U}^{\tilde{\tau}}[SuC,Sug]) \,\|$

$\tilde{2}$.g $\mathbf{X}^{\tau}[Fcen,Fr1]'' \times \{(\mathbf{\Omega}_{Fcen,N} \times \mathbf{U0}^{\tau p}(N,Fr1))''\}^{-1} \times (\mathbf{\Omega}_{Fcen,N} \times \mathbf{U0}^{\tilde{\tau} p}(N,Fr1)) \; //$

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[25] 'NAICS' stand for North American Industry Classification System (USDOC, Census Bureau, 2024).

[26] Any non-BEA data source that measures the same economic stock or flow as a BEA account of interest is given the same matrix or vector symbol with a '0' annotated, such as **U0** and **M0**, as a non-BEA measure of the BEA *Use* and *Make* tables respectively.

[27] From BEA's D-level nominal and real gross activity output data in 22.a, we derived F-level counterparts as $\mathbf{y}^{\tilde{\tau}}[FA] = \mathbf{\Omega}_{FA,DA} \times \mathbf{y}^{\tilde{\tau}}[DA]$, and $\mathbf{qy}^{\tilde{\tau}}[FA] = \mathbf{\Omega}_{FA,DA} \times \mathbf{qy}^{\tilde{\tau}}[DA]$, and in 22.b we used previous year volume matrix, $\mathbf{qZ}^{(\tilde{\tau}-1)}$.

[28] In equation $\tilde{7}$, $\mathbf{w}^{\tau(\tilde{\ })}[\lambda C \times O] = \mathbf{\Omega}_{\lambda C \times O,\lambda C} \times \mathbf{w}^{\tau(\tilde{\ })}[\lambda C]$, where $\lambda \in (F,Su)$.

$2^{\sim}$.h $\mathbf{X}^{\tau}[Fxcen,Fr1]'' \times \{(\mathbf{\Omega}_{Fxcen,SuC} \times \mathbf{U}^{\tau p}(SuC,\mathrm{Sur}1))''\}^{-1} \times (\mathbf{\Omega}_{Fxcen,SuC} \times \mathbf{U}^{\tau^{\sim} p}(SuC,Sur1))]$

$3^{\sim}$.a $\mathbf{L}^{\tau^{\sim}}[FIr2,FAC] = (\mathbf{L}^{\tau}[FI,FAC] \times [\mathbf{U0}^{\tau^{\sim} p}(FI,UsA) \times$

$3^{\sim}$.b $\{\mathbf{U0}^{\tau p}(FI,UsA)''\}^{-1} \| \mathbf{U0}^{\tau^{\sim} p}(FI,N) \times \{\mathbf{U0}^{\tau p}(FI,N)''\}^{-1}] \times \mathbf{\Omega}_{FUsAN,FA} \| 0[FI,FC]) //$

$3^{\sim}$.c $(\mathbf{L}^{\tau}[Fr2,FAC] \times [0[Fr2,FA] \| [-\mathbf{U0}^{\tau^{\sim} p}(N,Fr2)' \times \{-\mathbf{U0}^{\tau p}(N,Fr2)''\}^{-1} \|$

$3^{\sim}$.d $-\mathbf{U0}^{\tau^{\sim} p}(SuC,Fr2)' \times \{-\mathbf{U0}^{\tau p}(SuC,Fr2)''\}^{-1}] \times \mathbf{\Omega}_{NSuC,FC}]'')$

$7^{\sim}$.a $\mathbf{PCEb}^{\tau^{\sim}}[FC \times O,FE] = \mathbf{\Omega}_{FC \times O,FC \times O \times FE}$

$7^{\sim}$.b $\times [\{\mathrm{vec}(\mathbf{\Omega}_{FC \times O,SuC \times O} \times (\mathbf{w}^{\tau^{\sim}}[SuC \times O]'' \times \{\mathbf{w}^{\tau}[SuC \times O]''\}^{-1})$

$7^{\sim}$.c $\times \mathbf{PCEb}^{\tau}[SuC \times O,SuE] \times \mathbf{\Omega}_{SuE,FE})''\}^{-1}$

$7^{\sim}$.d $\times \mathrm{vec}((\mathbf{w}^{\tau^{\sim}}[FC \times O]'' \times \{\mathbf{w}^{\tau}[FC \times O]''\}^{-1}) \times \mathbf{PCEb}^{\tau}[FC \times O,FE])$

$7^{\sim}$.e $\times \mathrm{vec}(\mathbf{\Omega}_{FC \times O,SuC \times O} \times \mathbf{PCEb}^{\tau^{\sim}}[SuC \times O,SuE] \times \mathbf{\Omega}_{SuE,FE})]'' \times \mathbf{\Omega}_{FC \times O \times FE,FE}$

$7^{\sim}$.f $\times \mathbf{scale}^{\tau^{\sim}}(FE)''$

Equation $1^{\sim}$ reports how we compiled our initial transaction submatrix ($\mathbf{Z}^{\tau^{\sim}}$) in nonbenchmark years using current year *D*-level gross output and commodity price data aggregated to the *F* level, current year activity value added data in both *Us* and *N* levels aggregated to the *F* level,[29] and different sections of the current benchmark year constant price technology matrix. Recalling this is assumed to be a fixed technology matrix over the interbenchmark interval, our null hypothesis is the prior estimate for $\mathbf{Z}^{\tau^{\sim}}$ is unbiased and fully determined. The expression on the right of the equation in $1^{\sim}$.a inflates elements of the $\mathbf{q}'\mathbf{\Lambda}^{\tau\sim 1}$ previous year fixed technology matrix to current year values and compiles row shares for each element within this subsection through multiplying by the inverted diagonalized row total vector. Above we described the equation for this previous year row share submatrix, $\mathbf{q}'\mathbf{\Lambda}^{\tau\sim 1}[FA,FC]$ and hypothesized its time invariance from previous to current year. Because a nominal translation of this parameter is not time invariant, we inflated this parameter and translated to nominal row shares in lines $1^{\sim}$.a and $1^{\sim}$.b to distribute the observed current year gross activity outputs ($\mathbf{y}^{\tau^{\sim}}[FA]$) to each commodity produced for all activities. In $1^{\sim}$.b and $1^{\sim}$.c, the $[FC,FA]$ subsection of $\mathbf{q\Lambda}[FC, FA]$ is inflated to current year prices and column shares are compiled to again share out current year gross activity outputs net of gross activity payments to primary factors ($\mathbf{y}^{\tau^{\sim}}[FA]$ - $(\mathbf{\Omega}_{1,FI} \times \mathbf{L}^{\tau^{\sim}}[FI,FA])'$). This netting out of primary factor payments is necessary because intermediate direct requirement shares represent shares of total intermediate costs and do not include primary factor costs.

Equation $2^{\sim}$ describes how we compiled our initial injection submatrix ($\mathbf{X}^{\tau^{\sim}}$) in nonbenchmark years using an array of data sources outlined in equation lines $2^{\sim}$.b to $2^{\sim}$.h. For the household expenditure component of this submatrix, outlined in lines $2^{\sim}$.b to $2^{\sim}$.d, we needed different approaches for expenditures on consumer goods and services and on margin commodity expenditures. For goods and services, we summed the current nonbenchmark year $\mathbf{PCEb}^{\tau^{\sim}}$ commodity expenditure data in producer prices ($FC \times p$) across all expenditure categories (FE). The source data for this component is described in equation $7^{\sim}$ and discussed below. For the five transportation commodities, there is a one-to-one correspondence between the annually published *Su*-level use table and the target *F*-level account, $\mathbf{U}^{\tau^{\sim}}[Sutr,Suh] = \mathbf{X}^{\tau^{\sim}}[Ftr,SuFh]$. The wholesale and retail components of our target subaccount are described in lines $2^{\sim}$.c and $2^{\sim}$.d, respectively. Our approach for these was to produce current price weighted *F* level commodity budget shares of *Su* level aggregated commodities, which were used to share our current year published *Su*-level use table data, $\mathbf{U}^{\tau^{\sim}}[Suws,Suh]$ and $\mathbf{U}^{\tau^{\sim}}[Surt,Suh]$. With only one *Su*-level wholesale commodity, an

[29] The expression $\mathbf{L}^{\tau^{\sim}}[\ldots]$ in $1^{\sim}$.c is expanded in equation $3^{\sim}$.

aggregation matrix is not required to calculate *F*-level budget shares with the *Su*-level use table data (equation line 2˜.c). Several *Su*-level retail commodities require an aggregation matrix to concord *F*-level budget shares with the *Su* level use table data (equation line 2˜.d). An identical approach is applied to our estimation of all government expenditures (equation line 2˜.f). However, capital outlays (*Fk*) are historically volatile concerning annual published *Su*-level data on direct investment expenditures and year-to-year inventory change. Rather than carry forward current price weighted benchmark year investment budget shares, our approach was to apply benchmark-to-current-year published percentage change in nominal *Su*-level commodity investments to their corresponding benchmark year *F*-level investments (equation line 2˜.e).[30] Current year international exports *N*-level data are available from the Census Bureau ($\mathbf{U0}^{\tilde{\tau}p}(N,Fr1)$) with corresponding *F-l*evel coverage for a subset of commodities (*Fcen*). We applied benchmark-to-current-year published percentage change in these data to their corresponding benchmark year *F*-level values (equation line 2˜.g). Gaps in this data source for *F*-level coverage apply to a subset of commodities (*Fxcen*). For this subset we applied benchmark-to-current-year published percentage change in nominal *Su*-level commodity international export data to their corresponding benchmark year *F*-level values (equation line 2˜.h) using the method described in footnote 27.

Equation 3˜ describes how we compiled our initial leakage submatrix ($\mathbf{L}^{\tilde{\tau}}$) in nonbenchmark years using an array of data sources outlined in equation lines 3˜.a to 3˜.d. For institutional primary factor payments (*FI*), we have had two current year primary data sources, BEA *Su*-level data ($\mathbf{U0}^{\tilde{\tau}p}(FI,UsA)$), and Census Bureau *N*-level data ($\mathbf{U0}^{\tilde{\tau}p}(FI,N)$) that together cover all *F*-level commodities. We applied benchmark-to-current-year published percentage changes in nominal primary factor payment data to their corresponding benchmark year *F*-level values (equation lines 3˜.a and 3˜.b). For international commodity imports, we applied benchmark-to-current-year published percentage changes in nominal international imports data at both the *N*-level ($-\mathbf{U0}^{\tilde{\tau}p}(N,Fr2)'$) and *Su*-level ($-\mathbf{U0}^{\tilde{\tau}p}(SuC,Fr2)'$) to their corresponding benchmark year *F*-level values (equation lines 3˜.c and 3˜.d).

Equation 7˜ describes how we compiled our annual tier 0 **PCEb** account. Data for this account include benchmark year expenditure data converted to current year prices to produce current price weighted *F*-level commodity budget shares of *Su*-level aggregated commodities, which were used to share our current year published *Su*-level expenditure data. These estimates were then rescaled using detailed BEA annual **pce**(*DE*) data in market prices (see equation 17˜), which provided a nearly 1-to-1 ratio of expenditure category ($FE \times m$) market value data points to expenditure commodity producer value data points ($FC \times p$).

The benchmark year model constraints of equations 17 and 18 ensure published integer values are preserved in the constrained maximum likelihood model solution. These constraints do not apply in the nonbenchmark year model, so we redefine equation 17˜ for the **scale** equation and equation 18˜ to incorporate BEA's annual $\mathbf{pce}^{\tilde{\tau}}(DE)$ data into the nonbenchmark years data system:[31]

[30] For example, if current year farm commodity investment in the summary *Use* table are 20 percent higher than values in the most recent benchmark year summary *Use* table, all 10 current year *F*-level farm commodity investment prior estimates are 10 percent higher than observed benchmark year *F*-level values.

[31] Because BEA $\mathbf{PCEb}^{\tau}$ data are published in benchmark years (*T*) the incorporation of $\mathbf{pce}^{\tau}(DE)$ in the benchmark year system would be redundant.

17˜.a $\mathbf{scale}^{\tilde{\tau}}(FE) = \{[\ \mathbf{\Omega}_{1,FC\times O} \times (\mathbf{\Omega}_{FC\times O,FC\times O\times FE}$

17˜.b $\times [\{\text{vec}(\mathbf{\Omega}_{FC\times O,SuC\times O}\times(\mathbf{w}^{\tilde{\tau}}[SuC\times O]'' \times \{\mathbf{w}^{\tau}[SuC\times O]''\}^{-1})\times \mathbf{PCEb}^{\tau}[SuC\times O,SuE]\times \mathbf{\Omega}_{SuE,FE})''\}^{-1}$

17˜.c $\times\ \text{vec}((\mathbf{w}^{\tilde{\tau}}[FC\times O]'' \times \{\mathbf{w}^{\tau}[FC\times O]''\}^{-1})\times \mathbf{PCEb}^{\tau}[FC\times O,FE])$

17˜.d $\times\ \text{vec}(\mathbf{\Omega}_{FC\times O,SuC\times O}\times \mathbf{PCEb}^{\tau}[SuC\times O,SuE]\times \mathbf{\Omega}_{SuE,FuE})]'' \times \mathbf{\Omega}_{FC\times O\times FE,FE})\ ]''\}^{-1}$

17˜.e $\times [\mathbf{\Omega}_{FE,DE}\times \mathbf{pce}^{\tilde{\tau}}(DE)]$

18˜ $\mathbf{\Omega}_{FE,DE}\times \mathbf{pce}^{\tilde{\tau}}(DE) = (\mathbf{\Omega}_{1,FC\times O} \times \mathbf{PCEb1}^{\tilde{\tau}}[FC\times O,FE])'$

The right side of the equality in equation line 17˜.a combined with equation lines 17˜.b–17˜.d compiled a diagonalized and inverted summation of the right side of the equality in equation line 7˜.a combined with equation lines 7˜.b–7˜.d. This ensures the initial values for $\mathbf{PCEb}^{\tilde{\tau}}[FC\times O,FE]$ will reflect current year data in $\mathbf{pce}^{\tilde{\tau}}(DE)$.

Equation 19 can be restated with nonbenchmark year data and maximization of 19˜ subject to constraint equations 9˜–18˜ represents the constrained maximum likelihood (CML) solution to our target annual Ag-FEDS account.[32] This CML model routinely solves in all interbenchmark years. Diagnostics for model performance report statistics on the relative approximation error (Rabinovich, 2005) for each subaccount of our target $\mathbf{rSAM}^{\tilde{\tau}}[F,F]$ account over the timeseries. For each nonzero account element $\mathbf{rSAM}^{\tilde{\tau}}[f1,f2]\ \forall\ (f1,f2) \in F$, the relative approximation error (**RAE**) is defined as follows:

23. $\mathbf{RAE}(f1,f2) = \mathbf{\delta rSAM}^{\tilde{\tau}}[f1,f2] \times \{\mathbf{rSAM}^{\tilde{\tau}}[f1,f2]\}^{-1}$

Collectively the statistics reported in figure 4 provide a comprehensive test of bias and reliability of our CML estimation model. Because the eight subaccounts highlighted in figure 4 use different input data sources and estimation approaches, evaluation statistics are developed separately for each.

The box in the box and whisker (B&W) diagrams in figure 4 depict interquartile range (IQR) (Wilcox, 2017) of **RAE** values by subaccount, which measures range between the 25th (1st quartile or Q1) and 75th percentile (3rd quartile or Q3) of **RAE** observations when ordered from low to high values. With 50 percent of observations including the median observation contained within the IQR, values were identified as outliers if they are 1.5 times beyond either end of the IQR as depicted by the whiskers in figure 2. Each B&W diagram reports the low and high Q1 and Q3 values in callouts over each subaccount timeseries. Tables for each subaccount are reported on the right side of figure 4 and provide the RAE median values, the number of observations, and the percent of observations within the whiskers of each annual subaccount. Each table also reports mean absolute percentage errors (MAPE) over all observations annually. MAPE measures the weighted average absolute RAE values over all observations by subaccount.

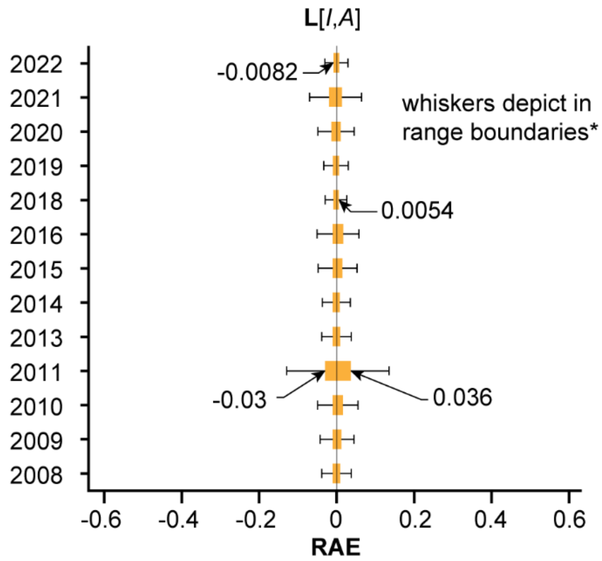
Collectively these RAE statistics provide compelling validation for the CML estimation model across all subaccounts and over the entire time series. Over 75 percent of the RAE distributions have median values of 0 measured out to four decimals, and over 95 percent have median values of 0 measured out to two decimals, indicating our source data are unbiased. The few nonzero median values show no relation to years removed from a benchmark, which speaks to our fixed technology transaction matrix null hypothesis. While there is evidence of increasing MAPE statistics with years beyond a benchmark, the RAE values are overwhelmingly within +/- 0.05.

[32] There are no endogenous variables in equation 17˜, rather it serves to compile the initial tier 0 value, $\mathbf{PCEb}^{\tilde{\tau}}$, and can be left out of the CML constraints in model specification.

In addition to these statistical properties, it is important to note that all relevant annual BEA data products are exactly replicated and represented in the target *F*-level **rSAM** and **PCEb** annual accounts. These include the annual *S*-level make, use, and **PCEb** tables, *Us*-level gross value added tables, and *D*-level gross activity output and **pce** tables.

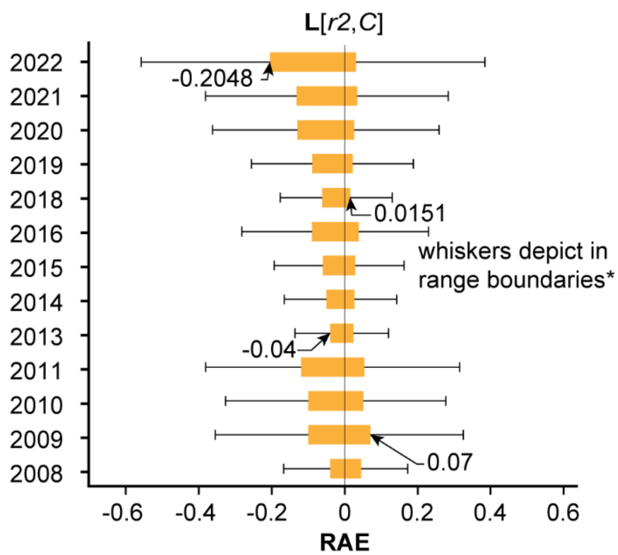
Figure 4 (1 of 4)

Relative approximation error box and whisker diagrams and summary statistics



Relative approximation error statistics for L[*I*,*A*] subaccount				
Year	RAE median	MAPE	Observations	Percent in range*
2022	0.000	0.22	351	84.6
2021	0.000	1.37	351	89.5
2020	0.000	1.01	351	85.5
2019	0.000	0.74	351	83.8
2018	0.000	0.45	351	90.6
2017	0.000	0.00	NULL	100.0
2016	0.000	0.75	354	92.1
2015	0.000	0.77	354	90.1
2014	0.000	0.66	320	88.1
2013	0.001	0.61	320	95.0
2012	0.000	0.00	NULL	100.0
2011	0.000	7.41	354	79.4
2010	0.000	0.85	354	83.9
2009	0.000	0.87	354	76.8
2008	0.000	0.57	354	84.2
2007	0.000	0.00	NULL	100.0

* In range defined as above Q1-1.5xIQR and below Q3+1.5xIQR



Relative approximation error statistics for L[*r2*,*C*] subaccount				
Year	RAE median	MAPE	Observations	Percent in range*
2022	-0.045	9.98	283	91.9
2021	-0.021	9.42	284	90.8
2020	-0.019	10.62	285	90.2
2019	-0.009	5.44	285	89.8
2018	-0.011	4.24	284	87.3
2017	0.000	0.00	NULL	100.0
2016	0.000	6.28	280	89.6
2015	0.000	4.71	280	89.6
2014	0.000	2.88	268	90.7
2013	0.000	2.57	267	91.0
2012	0.000	0.00	NULL	100.0
2011	0.000	6.53	281	92.2
2010	0.000	5.47	281	88.6
2009	0.000	6.27	281	92.5
2008	0.000	3.53	281	90.7
2007	0.000	0.00	NULL	100.0

* In range defined as above Q1-1.5xIQR and below Q3+1.5xIQR

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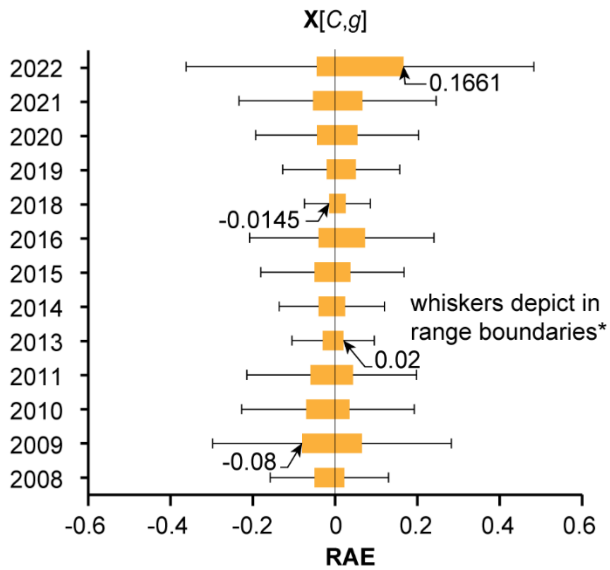
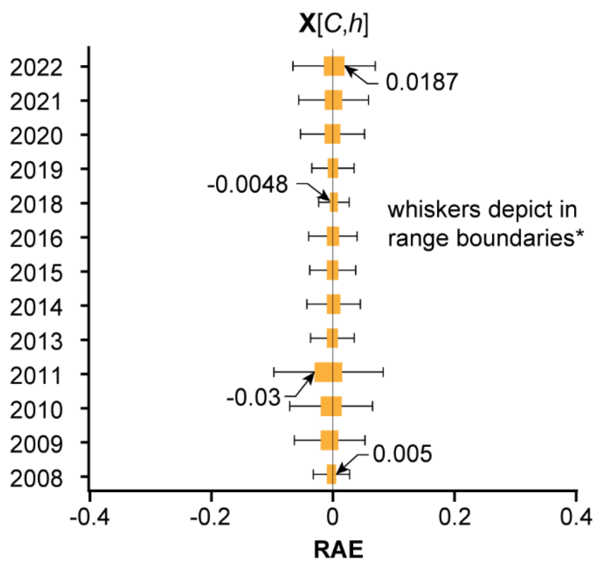
Figure 4 (cont., 2 of 4)

Relative approximation error box and whisker diagrams and summary statistics



Relative approximation error statistics for **X**[*C*,*g*] subaccount				
Year	RAE median	MAPE	Observations	Percent in range*
2022	0.016	4.23	117	93.2
2021	0.000	1.51	120	86.7
2020	0.000	0.77	120	86.7
2019	0.008	0.64	120	92.5
2018	0.001	0.33	120	87.5
2017	0.000	0.00	NULL	100.0
2016	0.003	1.79	117	88.9
2015	0.001	1.32	121	90.9
2014	0.000	0.74	117	92.3
2013	0.000	1.05	118	94.9
2012	0.000	0.00	NULL	100.0
2011	0.000	1.93	119	85.7
2010	0.000	1.87	119	87.4
2009	0.000	2.26	120	90.8
2008	-0.010	1.01	119	91.6
2007	0.000	0.00	NULL	100.0

* In range defined as above Q1-1.5xIQR and below Q3+1.5xIQR



Relative approximation error statistics for **X**[*C*,*h*] subaccount				
Year	RAE median	MAPE	Observations	Percent in range*
2022	0.000	2.68	242	76.9
2021	0.000	1.97	242	78.5
2020	0.000	2.45	242	79.3
2019	0.000	0.81	242	81.4
2018	0.000	0.64	242	81.0
2017	0.000	0.00	NULL	100.0
2016	0.000	1.81	245	80.8
2015	0.000	1.40	245	81.2
2014	0.000	1.00	209	86.1
2013	0.000	0.65	208	89.4
2012	0.000	0.00	NULL	100.0
2011	0.000	1.90	247	81.4
2010	0.000	1.64	247	79.8
2009	0.000	1.45	247	76.5
2008	0.000	0.94	247	76.5
2007	0.000	0.00	NULL	100.0

* In range defined as above Q1-1.5xIQR and below Q3+1.5xIQR

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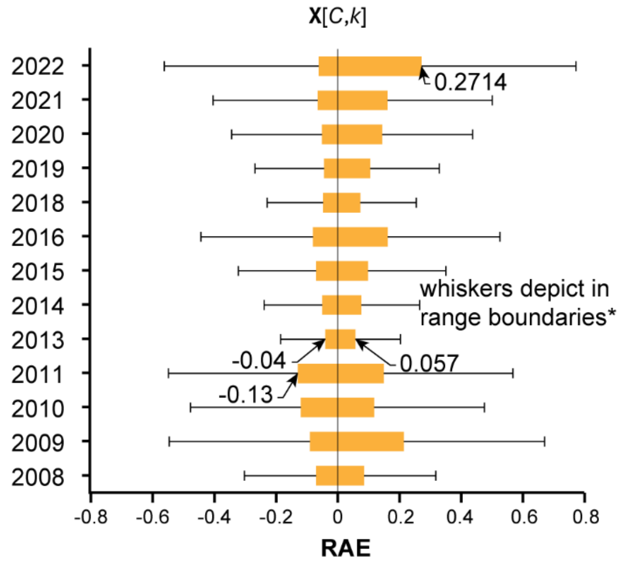
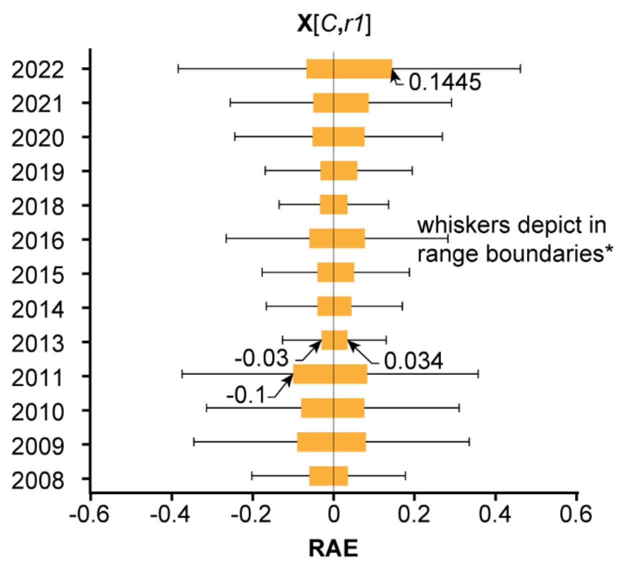
Figure 4 (cont., 3 of 4)

Relative approximation error box and whisker diagrams and summary statistics



Relative approximation error statistics for **X**[*C*,*k*] subaccount				
Year	RAE median	MAPE	Observations	Percent in range*
2022	0.057	13.13	297	90.2
2021	0.013	9.50	297	91.9
2020	0.018	5.67	297	87.5
2019	0.004	3.62	297	92.9
2018	0.004	2.21	297	93.9
2017	0.000	0.00	NULL	100.0
2016	0.011	9.90	299	94.3
2015	0.001	7.06	301	92.7
2014	0.004	5.09	294	93.5
2013	0.002	4.01	294	92.5
2012	0.000	0.00	NULL	100.0
2011	0.003	12.20	298	93.0
2010	0.000	11.85	299	92.0
2009	0.025	14.60	299	93.3
2008	0.000	8.89	296	93.9
2007	0.000	0.00	NULL	100.0

* In range defined as above Q1-1.5xIQR and below Q3+1.5xIQR



Relative approximation error statistics for **X**[*C*,*r1*] subaccount				
Year	RAE median	MAPE	Observations	Percent in range*
2022	0.007	6.54	317	92.7
2021	0.004	5.48	317	92.4
2020	0.001	5.34	317	91.5
2019	0.000	3.45	317	93.1
2018	0.000	2.19	317	91.8
2017	0.000	0.00	NULL	100.0
2016	0.000	6.17	316	92.7
2015	0.000	4.62	316	89.2
2014	0.000	3.22	295	91.9
2013	0.000	2.94	297	93.6
2012	0.000	0.00	NULL	100.0
2011	0.000	11.08	316	88.6
2010	0.000	9.53	316	86.7
2009	0.000	9.79	316	87.7
2008	0.000	9.34	316	86.4
2007	0.000	0.00	NULL	100.0

* In range defined as above Q1-1.5xIQR and below Q3+1.5xIQR

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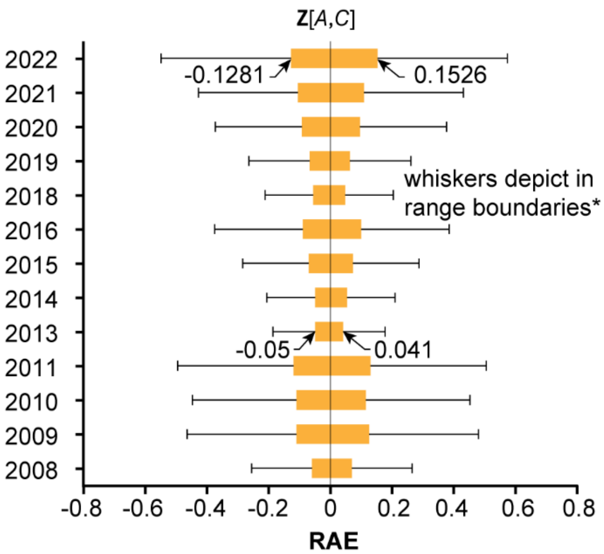
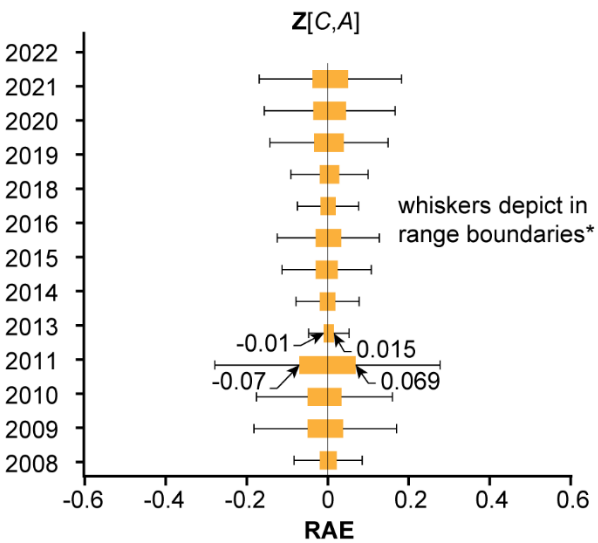
Figure 4 (cont., 4 of 4)

Relative approximation error box and whisker diagrams and summary statistics



Relative approximation error statistics for **Z**[*A*,*C*] subaccount				
Year	RAE median	MAPE	Observations	Percent in range*
2022	0.000	1.42	4,630	89.1
2021	0.000	0.98	4,568	91.2
2020	0.000	0.99	4,464	89.4
2019	0.000	0.57	4,568	91.7
2018	0.000	0.44	4,460	90.7
2017	0.000	0.00	NULL	100
2016	0.000	0.99	4,769	91.1
2015	0.000	0.81	4,792	91.1
2014	0.000	0.61	4,713	90.9
2013	0.001	0.47	4,696	93.0
2012	0.000	0.00	NULL	100.0
2011	0.000	1.70	4,987	91.9
2010	0.000	1.34	4,991	92.2
2009	0.000	1.26	4,990	92.0
2008	0.000	0.70	5,032	92.7
2007	0.000	0.00	NULL	100.0

* In range defined as above Q1-1.5xIQR and below Q3+1.5xIQR



Relative approximation error statistics for **Z**[*C*,*A*] subaccount				
Year	RAE median	MAPE	Observations	Percent in range*
2022	0.002	4.11	37,669	92.2
2021	0.002	3.77	37,149	93.2
2020	0.001	3.42	36,508	90.6
2019	0.002	2.16	37,314	91.8
2018	0.001	1.69	37,411	90.9
2017	0.000	0.00	NULL	100.0
2016	0.000	3.34	37,587	91.1
2015	0.000	2.93	37,742	91.2
2014	0.000	2.11	37,705	92.6
2013	0.001	1.67	37,461	90.5
2012	0.000	0.00	NULL	100.0
2011	0.000	9.56	37,827	90.1
2010	0.000	3.75	37,435	93.0
2009	0.000	3.51	37,021	92.6
2008	0.000	2.17	38,018	90.5
2007	0.000	0.00	NULL	100.0

* In range defined as above Q1-1.5xIQR and below Q3+1.5xIQR

RAE = Relative approximation error. MAPE = Mean absolute percentage error. IQR = Interquartile range. Q1 = Quarter 1. Q3 = Quarter 3.

Note: **L**[*I*,*A*] =intersection of institutional rows and activity columns of leakage submatrix; **L**[*r2*,*C*] =intersection of import row and commodity columns of leakage submatrix; **X**[*C*,*g*] =intersection of commodity rows and government column of injection submatrix; **X**[*C*,*h*] =intersection of commodity rows and household column of injection submatrix; **X**[*C*,*k*] =intersection of commodity rows and savings/invest column of injection submatrix; **X**[*C*,*r1*] =intersection of commodity rows and export column of injection submatrix; **Z**[*A*,*C*] =intersection of activity rows and commodity columns of endogenous transactions submatrix; **Z**[*C*,*A*] =intersection of commodity rows and activity columns of endogenous transactions submatrix; rectangles in box and whisker charts depict the range between first and third quarters of RAE estimates when ordered from low to high values; whiskers in box and whisker chart depict ranges between Q1-1.5x(Q3-Q1) and Q1 (left whisker) and between Q3 and Q3+1.5x(Q3-Q1) (right whisker); NULL indicates row statistics deducted without observations.

Source: USDA, Economic Research Service.

We also updated the hybrid margin accounts, $\mathbf{rSAM}^{\tau\tilde{}\lambda}[FC,FAI]$ for $\lambda \in (Setr, Sews, Sert, m)$. To conserve on notation, we described our simplified CML model in this step. For this update, we translated the most recent benchmark year margin plus market price accounts to current year prices then convert all margin and market price transactions to their share of the corresponding benchmark year producer price transactions also inflated to current year prices. These margin shares are applied to the $\mathbf{rSAM1}^{\tau\tilde{}}[FC,FAI]$ subaccount produced by the current year CML model described above to produce initial ill-posed current year margin and market price accounts. The simplified CML model produces well-posed margin accounts and has a weighted least square objective function that minimizes changes to the initial ill-posed margin accounts subject to:

$$\mathbf{\Omega}_{1,F\lambda} \times \mathbf{rSAM1}^{\tau\tilde{}}[F\lambda,FAI] = \mathbf{\Omega}_{1,FC} \times \mathbf{rSAM}^{\tau\tilde{}Se\lambda}[FC,FAI] + \mathbf{\Omega}_{1,F\lambda} \times \mathbf{rSAM}^{\tau\tilde{}m}[F\lambda,FAI],\ \lambda \in (tr,ws,rt)$$

These three constraints ensured the updated margin accounts are well-posed. Specifically, that each activity and institutional total outlay on margin commodities at producer prices (left side of above equations) equals their corresponding total margin outlays for all transactions plus direct expenditures on margin commodities at market prices (right side of above equality), when both sides are aggregated to their single sector level margin commodity value. The importance of carrying forward the margin accounts to nonbenchmark years will be evident in the next subsection.

3.2 Address Comingling of Commodity Flows

Next, we turned to our reorganization of the accounts, which focuses exclusively on the food economy. In the process we (1) addressed accounting step omissions in our source data; (2) adapted the U.S. Department of Commerce, Bureau of Economic Analysis (BEA) method of account redefinitions with a focus on the food economy; (3) decoupled and disaggregated food and beverage commodities that are locked into the foodservice and food-related activity technologies; and (4) implemented several fine tuning techniques.

To motivate this reorganization, consider that 57 percent of the $2.9 trillion in 2023 food and beverage spending in the United States represents spending on foodservices and the food and beverage commodities acquired either with foodservices or provided by employers or institutions (USDA, ERS, 2024a). We refer to this as food and beverages away from home (FBAFH). Accounting for this FBAFH spending within the **rSAM** accounting framework, all food and beverage commodities linked to FBAFH spending are embedded within the endogenous transactions subaccount ($\mathbf{Z}[FAC,FAC]$), and so are locked into fixed technology structure of these accounts. By comparison, the remaining food and beverage spending purchased at grocery stores or other points of purchase for home consumption are measured as individual final market commodity purchases by or for U.S. households. We denote this as food and beverages at home spending, or FBAH. What this means in terms of being able to account for food and beverage commodity flows linked to annual domestic food spending is that all FBAH spending can be examined for individual commodities such as fresh fruits and processed dairy products, whereas all FBAFH spending must be measured as bundle such as annual meals purchased at limited foodservice restaurants, or annual school meal purchases. As we will show, representing the movement of food commodities along FBAFH supply chains can become misdirected to incorrect supply chains in the multiplier model process. As a result, the translation of consumer food and beverage expenditures into measures of gross domestic outputs plus imports linked to accommodating these consumer expenditures become unreliable when expenditures are broken out into more detailed expenditure categories.

A small but effective example of this issue concerns the value of alcohol in purchased meals. This expenditure is measured in $\mathbf{PCEb}^{\tau}[DC \times p, DE]$ published for all benchmark years, including the 2017 benchmark year table (USDOC, BEA, 2024). Table 3 reports consumer expenditures on alcohol in purchased meals broken out by point of purchase and the producer value for beverage sales induced by these point-of-purchase consumer expenditures, both in dollars and as a share of consumer expenditures. This table highlights three accounting practices that can misdirect flows.

Table 3

Consumer expenditures on alcohol in purchased meals by marketing channel and producer value of beverages sold, 2017

Point of purchase	Consumer expenditures	Producer value of beverages sold				Producer share of consumer expenditures
		Beer	Wine	Spirits	Total	
	Million dollars					Percent
Air transportation	1,109.00	0.72	1.83	0.18	2.74	0.25
Rail transportation	20.00	0.00	0.00	0.00	0.00	0.01
Performing arts companies	33.00	0.01	0.00	0.00	0.02	0.05
Spectator sports	108.00	0.04	0.02	0.01	0.07	0.06
Promoters of performing arts and sports and agents for public figures	274.00	0.14	0.03	0.03	0.20	0.07
Full-service restaurants	61,725.00	314.63	230.82	805.47	1,350.92	2.19
Limited-service restaurants	5,520.00	39.77	24.82	14.47	79.07	1.43
All other food and drinking places	30,402.00	115.93	409.40	55.05	580.38	1.91

Source: USDA, Economic Research Service using U.S. Department of Commerce, Bureau of Economic Analysis, PCE Bridge Table data and Industry-By-Commodity Total Requirements table data.

Accounting Step Omissions

All **PCEb** foodservice expenditure categories identifying nonfoodservice and nonfood commodity expenditures (denoted *nfs*) must be reclassified as limited foodservice (as indicated by the make table) and all relevant elements of the target **PCEb2** and **rSAM2** accounts must adjust to reflect this reclassification:

- Equations 24–27 add the reclassified outlays reported in the first five data lines in table 3 as consumer expenditures on alcohol in purchased meals to purchases at limited-service restaurants in the PCE column of the **rSAM2** account and deducts this same amount from the reclassified rows of this same column, and do the same for the **PCEb2** account.
- In equations 28 and 29, the primary production of the misclassified *nfs* commodities (non-foodservice commodities that are incorrectly sold as a foodservice) is reclassified as secondary production of limited foodservice by the *Anfs* activities (activities producing the *nfs* commodities) and primary production of *nfs* is scaled back by same amount.

- The movement of outlays between commodities in the PCE column (*XFh*) of **rSAM2** and the adjustments of primary and secondary production among *Ansf* activities in the **rSAM2** preserve the account balance, whereas the similar adjustments to the **PCEb1** account preserve the full integration of the **rSAM2** and **PCEb2** accounts.

In the following equations, we used numbers after **PCEb** and **rSAM** to indicate intermediate matrices while preparing the final matrices presented in figure 5. One accounting practice involved the recording of expenditure outlays on commodities that are coproducts with commodities the outlay was intended to represent. For the example in table 3, we found that the first five expenditure lines (i.e., air and rail freight, performing arts, spectator sports, and promoters) were listed as commodities consumed as alcohol in purchased meals. This appeared to be an accounting step omission since transportation and spectator services are activities that jointly produce small amounts of foodservice commodities. For example, in the 2017 BEA make table ("Make Table, Before Redefinitions, 2017" from USDOC, BEA, 2024a) we found nearly $3.5 billion of the limited-service restaurants commodity is produced by the spectator services activities, and yet small amounts of alcohol are recorded being consumed as spectator and transportation services.[33] Although such omissions are minor in the BEA accounts, they illuminate the misdirection of flows in applying multiplier analysis. For example, in table 3 we saw that a multiplier calculation indicates between a 0-cent and 0.2-cent (2.74/1,109) producer value of all alcoholic beverages sold per consumer dollar spent. This was a result of translating the alcohol purchases in the form of commodities listed in the first five data lines of table 3 into the total requirements of beer, wine, and spirits to accommodate demand for these nonfoodservice (*nfs*) commodities purchased for alcoholic beverages. But the activities producing the *nfs* (*Anfs*) commodities purchase little if any alcoholic beverages according to their per unit direct requirements as reported in $\mathbf{\Lambda}^{\tau}$ (equation 20). Included in our nonfoodservices category are all the activities with primary outputs that are the nonfoodservice commodities.

However, roughly $3.5 billion of output for the activities producing these commodities (*Anfs*) was identified as joint production of limited-service restaurants' (*a342*) (for more information, see supplemental table A.1). This suggests that the outlays listed in the first five data lines of table 3 should be reclassified as purchases of limited-service restaurants[34] by households (the *XFh* column in **rSAM2** and the *Xf2202* column in **PCEb2**) (for more information, see table 5 below) in both tier 0 accounts.

24. $\mathbf{rSAM2}^{\tilde{\tau}}(c342,Fh) = \mathbf{rSAM1}^{\tilde{\tau}}(c342,Fh) + \mathbf{\Omega}_{1,nfs\times p} \times \mathbf{PCEb1}^{\tilde{\tau}}[nfs\times p,Xf2202]$

25. $\mathbf{rSAM2}^{\tilde{\tau}}(nfs,Fh) = \mathbf{rSAM1}^{\tilde{\tau}}(nfs,Fh)$ **-** $\mathbf{PCEb1}^{\tilde{\tau}}[nfs\times p,Xf2202]$

26. $\mathbf{PCEb2}^{\tilde{\tau}}(c342\times O,Xf2202) = \mathbf{PCEb1}^{\tilde{\tau}}(c342\times O,Xf2202) + \mathbf{\Omega}_{O,nfs\times O} \times \mathbf{PCEb1}^{\tilde{\tau}}[nfs\times O,Xf2202]$

27. $\mathbf{PCEb2}^{\tilde{\tau}}[nfs\times O,Xf2202] = 0$

The commodity assembly subaccount is also updated to reflect this new joint production:

28. $\mathbf{rSAM2}^{\tilde{\tau}}(Anfs,c342) = \mathbf{rSAM1}^{\tilde{\tau}}(Anfs,c342) + \mathbf{PCEb1}^{\tilde{\tau}}[nfs\times p,Xf2202]$

29. $\mathbf{rSAM2}^{\tilde{\tau}}(Anfs,nfs) = \mathbf{rSAM1}^{\tilde{\tau}}(Anfs,nfs)$ **-** $\mathbf{PCEb1}^{\tilde{\tau}}[nfs\times p,Xf2202]$

[33] BEA does not allocate any transportation activity outputs to foodservice, opting to instead treat food and beverage consumption on travel as transportation commodity consumption, even as some of this is classified as food and beverage consumption in the $\mathbf{PCEb}^{\tau}$ table. The same is true for the motion picture activity, which is treated as a food consumption commodity for meals at other eating places.

[34] The updated intermediate accounts developed in this section, **rSAM2** and **PCEb2** (and all subaccounts), have built on **rSAM1** and **PCEb1** accounts developed in the previous section. We used the nonbenchmark year superscript in all subsequent equations because there are more of them, but all equations also apply to benchmark years.

Only a subset of the *Anfs* commodities is recorded as food or beverage commodity outlays, but this subset will be grouped with the larger set-in steps discussed below such that a special designation for this subset is unnecessary.

Redefinitions

All production of a foodservice commodity (*fs*) by any nonfoodservice activity (*Anfs*) in **rSAM2** is redefined as primary production by the corresponding *Afs* activity, and in the process the unique direct intermediate and primary factor requirements for production by all corresponding *Afs* are moved from the *Anfs* activities over to the *Afs* activities. If not all of the required foodservice inputs were available for movement by any *Ansf* activity, a maximum likelihood model is specified to reallocate all surplus inputs among the *Ansf* activities subject to reclassification to all input deficit activities with offsetting inputs flowing back and changes minimized to the structure of all *Ansf* technologies as defined by the direct requirement matrix:

- Equation 30 defines a modified and augmented direct requirement submatrix for *Afs* activities that (1) preserves the total volume of all caloric (*fnb*) input requirements but reflects the mix of specific caloric inputs that exist in the *Anfs* activites; (2) preserves the values of all non-caloric intermediate inputs of the *Afs*; and (3) adds direct per unit primary input requirements of the *Afs* activities to this modified and augmented submatrix (**ΛΛ**).
- Equations 31–34 extract and reallocate all *Afs* inputs between *Anfs* and *Afs*, then redefine all secondary *fs* production by *Anfs* activities as additional primary production by *Afs* activities.
- Equations 35–44 (1) extract all rows and columns from **rSAM2** relevant to the CML model into an initial unbalanced submatrix (**QP0**) and introduce a target balanced variable submatrix (**QP1**) and change parameter (**δQP**); and (2) introduce a series of model constraints to ensure subaccount balance, preserve initial values not subject to change, and introduce the maximum likelihood equation (equation 40).
- Resulting model estimates, **QP1**, achieve the full redefinition objectives of this section and results are integrated back into the target **rSAM2** account (equation 45).

A second problematic accounting practice table 3 highlights relates to how production is measured for multi-product activities. Remaining *Anfs* activities that produce one or more types of foodservices are transferring similar nonfoodservice technologies to the foodservice activities. For example, the $1.7 billion of limited-service restaurants output by the hospitals activity (for more information, see "Make Table, Before Redefinitions, 2017" from USDOC, BEA, 2024a) results in a $1.7 billion pool of foodservices being produced by the same technology that provides hospital services. This $1.7 billion pool by itself is a very small fraction of the roughly $1 trillion in foodservice activity outputs. Beyond hospital activities, other foodservice outputs by all the nonfoodservice activities reported in the BEA make table produces well over 10 percent of total foodservices annually, which is not trivial. This 10+ percent misclassification will misdirect substantial commodity flows of a multiplier analysis in a similar manner as is revealed by the severely understated producer share measures reported in table 3.

To address this issue, we employed an accounting technique used by BEA that they refer to as redefinitions.[35] This involved the allocation of secondary products and their associated inputs within both make and use tables such that they were reassigned to the industry in which they are the primary products (USDOC, BEA, 2011).

[35] We were not able to work with the BEA redefinitions because all key input data for estimating our annual *F* accounts, including gross industry output and value added, were only available for the before redefinition accounts.

Our approach to redefinition was to employ a simple mathematical programing model that extracts the foodservice inputs including primary factors from the nonfoodservice activities. We started by adapting the foodservices direct requirement submatrix to reflect the food and beverage (*fnb*) commodity mix of the pooled *Anfs* activities.

30.a $\mathbf{\Lambda\Lambda}^{\tilde{\tau}}(\mathit{FCL,Afs}) = \mathbf{\Lambda}^{\tilde{\tau}}(\mathit{FC,Afs})\ //\ (\ \mathbf{rSAM2}^{\tilde{\tau}}(\mathit{FL,Afs}) \times \{(\mathbf{\Omega}_{\mathit{1,FCL}}\times\mathbf{rSAM2}^{\tilde{\tau}}(\mathit{FCL,Afs}))''\}^{-1}\ )$

30.b $\mathbf{\Lambda\Lambda}^{\tilde{\tau}}(\mathit{fnb,Afs}) = \{\mathbf{\Omega}_{\mathit{1,fnb}}\times\mathbf{rSAM2}^{\tilde{\tau}}(\mathit{fnb,Anfs})\times\mathbf{\Omega}_{\mathit{Anfs,1}}\}^{-1}\times(\mathbf{rSAM2}^{\tilde{\tau}}(\mathit{fnb,Anfs})\times\mathbf{\Omega}_{\mathit{Anfs,1}})\times\mathbf{\Omega}_{\mathit{1,fnb}}\times\mathbf{\Lambda}^{\tilde{\tau}}(\mathit{fnb,Afs})$

In 30.b, we ensured that the total direct requirement of caloric inputs for the existing foodservice activities ($\mathbf{\Omega}_{\mathit{1,fnb}}\times\mathbf{\Lambda}^{\tilde{\tau}}(\mathit{fnb,Afs})$) is preserved, but the product mix among *fnb* commodities reflected those of the pooled nonfoodservice activities (*Anfs*).

Next, we extracted these adapted foodservice production technologies from the targeted nonfoodservice activities and add these purchased inputs to the foodservice activities:

31. $\mathbf{rSAM2}^{\tilde{\tau}}(\mathit{FCL,Anfs}) = \mathbf{rSAM2}^{\tilde{\tau}}(\mathit{FCL,Anfs})\ \mathbf{-}\ \mathbf{\Lambda\Lambda}^{\tilde{\tau}}(\mathit{FCL,Afs}) \times (\mathbf{\Omega}_{\mathit{Afs,fs}}\times\mathbf{rSAM2}^{\tilde{\tau}}(\mathit{Anfs,fs})')$

32. $\mathbf{rSAM2}^{\tilde{\tau}}(\mathit{FCL,Afs}) = \mathbf{rSAM2}^{\tilde{\tau}}(\mathit{FCL,Afs}) + \mathbf{\Lambda\Lambda}^{\tilde{\tau}}(\mathit{FCL,Afs}) \times [\mathbf{\Omega}_{\mathit{Afs,fs}}\times\mathbf{rSAM2}^{\tilde{\tau}}(\mathit{Anfs,fs})'\times\mathbf{\Omega}_{\mathit{Anfs,1}}]''$

With this redefinition we must update the assembly of the *fs* and *nfs* commodities:

33. $\mathbf{rSAM2}^{\tilde{\tau}}(\mathit{Afs,fs}) = \mathbf{rSAM2}^{\tilde{\tau}}(\mathit{Afs,fs}) + [\mathbf{\Omega}_{\mathit{1,Anfs}}\times\mathbf{rSAM2}^{\tilde{\tau}}(\mathit{Anfs,fs})]''$

34. $\mathbf{rSAM2}^{\tilde{\tau}}(\mathit{Anfs,fs}) = 0$

At this point our $\mathbf{rSAM2}^{\tilde{\tau}}$ account is balanced. However, it is possible that some elements from one or more of the nonfoodservice activities from which we extracted purchased foodservice inputs had a deficit of some inputs. This would be reflected by negative outlays for those deficit scenarios. We know that collectively these activities will have a net zero outlay on purchased food and beverage commodities (*fnb*) since the total outlay extracted from these activities was scaled to ensure all *fnb* commodities were reclassified as foodservice inputs and the remaining reclassified inputs are scaled to reflect the foodservices technologies required to prepare and service these caloric inputs. This means that all deficits of *fnb* commodity outlays among a subset of *Anfs* activities are exactly offset by surpluses from the remaining subset of activities. For noncaloric inputs, we could not rule out a net deficit of all inputs across *Anfs* activities, so we must account for this unlikelihood (equation 42) to ensure feasibility of our constrained weighted least square quadratic program (QP) to minimally adjust outlays among *Anfs* activities while preserving account balance:

Define subaccount boundaries, parameters, and variables:

35. $\mathbf{QP0}(\mathit{FCL,AfsAnfs}) = \mathbf{rSAM2}^{\tilde{\tau}}(\mathit{FCL,AfsAnfs})$ (parameter)

36. $\mathbf{QP1}(\mathit{FCL,AfsAnfs})$ (variable)

37. $\mathbf{\delta QP}(\mathit{FCL,AfsAnfs}) = (\mathrm{vec}[\mathbf{QP1}(\mathit{FCL,AfsAnfs})\ \mathbf{-}\ \mathbf{QP0}(\mathit{FCL,AfsAnfs})])''$ (variable)

Declare model constraints and the QP objective function:

38. $\mathbf{QP1}(\mathit{FCL,AfsAnfs}) \times \mathbf{\Omega}_{\mathit{AfsAnfs,1}} = \mathbf{QP0}(\mathit{FCL,AfsAnfs}) \times \mathbf{\Omega}_{\mathit{AfsAnfs,1}}$

39. $\mathbf{\Omega}_{\mathit{1,FCL}} \times \mathbf{QP1}(\mathit{FCL,AfsAnfs}) = \mathbf{\Omega}_{\mathit{1,FCL}} \times \mathbf{QP0}(\mathit{FCL,AfsAnfs}) \times \mathbf{\Omega}_{\mathit{AfsAnfs,1}}$

40. $\mathrm{Max}[\mathscr{L}2(\mathbf{QP1})] = -0.5 \times [\mathbf{\delta QP}(\mathit{FCL,AfsAnfs}) \times (\mathrm{vec}[\mathbf{vQP0}(\mathit{FCL,AfsAnfs})])^{-1} \times \mathbf{\delta QP}(\mathit{FCL,AfsAnfs})]$

If, as we did for equation 19, we assumed the unreported coefficients of variation among **QP0** elements are the same (all initial values were equally reliable), we could use the reported **QP0** as linear unbiased transformations of their variance matrices, and equation 40 is the likelihood equation. Even when initial values are not equally reliable, equation 40 is a weighted least square solution which minimizes adjustments to initial estimates subject to constraints included in equations 38 and 39. This model routinely solves[36] in all years as expected since initial deficit elements (negative values) were usually limited to the caloric inputs that have a zero-sum deficit as discussed above. The following model constraints were added to fine tune the model and account for outlier scenarios when the model is applied over all annual accounts:

41. $\mathbf{QP1}(FL,AfsAnfs) = \mathbf{QP0}(FL,AfsAnfs)$ (fix all outlays on primary factors at their initial values)

42. $\mathbf{QP1}(FC,AfsAnfs) \geq 0$ (outlays on all commodities are non-negative)

43. If $\mathbf{QP0}(\lambda 1,\lambda 2) = 0$, then $\mathbf{QP1}(\lambda 1,\lambda 2) = 0, \forall \lambda 1 \in FC \;\&\; \lambda 2 \in AfsAnfs$ (fix all prior zeros)

44. If $\mathbf{QP0}(\lambda,Anfs) \times \mathbf{\Omega}_{Anfs,1} \geq 0$, then $\mathbf{QP1}(\lambda,Afs) = \mathbf{QP0}(\lambda,Afs), \forall \lambda \in FCL$

Equation 44 is included in the unlikely event that the collective outlays of *Anfs* activities on any noncaloric foodservice input did not cover the amount needed for reclassification of the joint foodservice outputs of these activities. Should a deficit occur, the model allows for minimal adjustments of these inputs away from *Afs* activities to remove such deficits. The resulting output is inserted into the tier 0 account:

45. $\mathbf{rSAM2}^{\tilde{\tau}}(FCL,AfsAnfs) = \mathbf{QP1}(FCL,AfsAnfs)$

For context, a multiplier analysis based on the final Ag-FEDS account for 2017 produces a measure for the alcoholic beverage producer share of consumer spending on alcohol in purchased meals of 36.5 percent overall. This share is only obtained through full (29.0 percent) foodservice marketing channels, limited (45.7 percent) foodservice marketing channels, and through other food and drinking places (46.9 percent).

Decoupling and Disaggregation

Added dimensions are introduced (table 4) to the *F* accounts in a new **rSAM3**[37] account to facilitate decoupling and disaggregation of all caloric commodities (*cal*) used by all foodservice activities (*Afs*). Expanded dimensions include additional activities and commodities covering the caloric commodities handled by foodservices (*Afsc,fsc*), plus redefined activities and commodities now representing only foodservices apart from the food and beverages being served (*Afs,fs*):

- Equation 46 maps out the new **rSAM3** and **PCEb3** accounts of dimension *F2*[38] and inserts the entire content of **rSAM2** and **PCEb2** within this larger dimension account, while equation 47 moves the caloric commodities (*cal*) from the *Afs* activities to the *Afsc* activities.
- Equations 48–50 reallocate the assembly of a portion from the *Afs* over to the new *fsc* commodities to preserve the balance of supply and use of each commodity.
- Equations 51–53 split outlays on *fs* measured in **rSAM2** to split outlays on the new *fsc* commodities and the redefined *fs* commodities, both for foodservice activities (*Afs*) in equation 52–53, and for households (*Fh*) in equations 52h−53h.
- All remaining activity outlays on foodservices are split using a voucher accounting approach discussed later.

[36] We solve as a quadratically constrained programming (QCP) problem using GAMS version 36.1 (GAMS, n.d.) and the CPLEX solver.

[37] Each iteration of our target **rSAM** and **PCEb** accounts (4 iterations in total) bring us closer to our final specification. It is convenient for mathematical specifications to sometimes refer to previous iterations when defining changes.

[38] Some equations make it convenient to distinguish updated dimensions such as are introduced in table 4 so we use an alias, *F2*, when introducing new dimensions and revert to *F* in subsequent equations.

- The steps summarized here expand the dimensions of our target **rSAM** and **PCEb** accounts and populate some of the new activity and commodity rows with redirected expenditures.

There remains a large amount of intermediate food and beverage commodity outlays by many activities as well as foodservice outlays as a business expense of many activities. To disentangle and accurately account for these food and beverage expenditures, a voucher accounting system is introduced in equations 54–73:

- Intermediate outlays on vouchers (*v01–v05*) replace outlays on foods and beverages (*fnb*) for all activities, and five categories of vouchers procure all *fnb* (food and beverage commodities more narrowly defined for non-foodservice activity purchases than the *cal* set used for foodservices) and *fs* (foodservices) for the various activities:
  - *v01* vouchers are for meals at work and replace all activity outlays on foodservices;
  - *v02* vouchers are for employer furnished food and beverages and replace *fnb* outlays for activities *Aμv02*;
  - *v03* vouchers are institutional furnished meals and food assistance, replacing *fnb* outlays for activities *Aμv03*;
  - *v04* vouchers are for school furnished meals replacing *fnb* outlays for activities *Aμv04*; and
  - *v05* vouchers are for food and beverage R&D and replace *fnb* outlays for activities *Aμv05*.
- Equations 54–56 replace *fnb* outlays among *Aμv02* to *Aμv05* activities (see table A.2 for descriptions of all tier 2 and 3 sets) with their corresponding vouchers and adds this *fnb* procurement to the corresponding voucher accounts (*v02–v05*).
- Equation 57–59 assign voucher overhead costs for labor (*vo1*) among *Aμv02* and *Aμv04* activities as a share of *v02* and *v04* outlays measured in equation 55. The share equals the ratio of labor to caloric costs for limited foodservice activities, then deducts labor outlays by same amount among same activities, and moves these outlays to the *vo1* voucher account.
- Equations 60–68 measure all margin costs (e.g., transportation, wholesale, retail) for *fnb* procurement among voucher *v02* to *v05* accounts and (1) replaces these outlays among *Aμv02* to *Aμv05* activities with additional voucher outlays, and (2) move this replaced margin cost to corresponding voucher accounts.
- Equations 69–73 reclassify activity outlays on foodservices (except foodservice activities which were dealt with above) to outlays on voucher *v01*. These equations consolidate all reclassified foodservice outlays over to the *v01* voucher account, then split these outlays between *fs* and *fsc* commodities in the same manner as done above for households and *Afs* activities.

Referring to table 3, a third accounting practice that was problematic for multiplier model applications that was highlighted in the table relates to how food and beverage commodities are locked into the foodservice technologies with whom they are sold and so must be decoupled. To explain, we first note that our redefinitions outlined in equations 24–45 will have the effect of increasing the alcoholic beverage producer share calculation reported in the far-right column of table 1 to ranges slightly higher than the 1.4–2.2 percent range reported in the last three data lines. Slightly higher because, as mentioned above, the *Anfs* activities account for over 10 percent of gross output for the foodservice commodities and with our redefinitions we extract the foodservice technologies (*Afs*) that table 3 demonstrate reflect a much higher producer share. Before redefinitions, the *Anfs* technologies are extracted. Our redefinitions impact far more than the alcohol in purchased meals highlighted in table 3, such as purchased meals and nonalcoholic beverages. Yet even after these redefinitions, it may appear surprising to many that alcoholic beverage producers sell $1–$2 of product for every $100 spent on their products by customers at bars and restaurants. In fact, this calculation is far too low, and this is because, as just mentioned, food and beverage commodities are locked into the foodservice technologies with whom they are sold. Thus the alcoholic beverages are sold as a package deal with all other caloric commodities to the alcohol customers at these restaurants and bars.

Our approach to address this locked-in technology problem was to decouple and disaggregate all caloric commodity flows. Here we started with the foodservices technology (*Afs*), where we decoupled the caloric commodity inputs (*cal*) from foodservices.[39] The previously defined *fnb* commodity set is the complete set of caloric inputs for nonfoodservice activities producing foodservices. These are different than *cal* both because they were organized differently and because the commodities identified in *fnb* were a subset of *cal* due to difficulties defining some inputs as caloric in the former set.

Table 4

Decoupled caloric activities, commodities, and vouchers added to the reduced social accounting matrix (rSAM)

Activity	Commodity	Description
a358	*c358*	Food and soft drinks at full-service restaurants
a359	*c359*	Food and soft drinks at limited-service restaurants
a360	*c360*	Food and soft drinks at all other food and drinking places
a361	*c361*	Alcoholic beverages at full-service restaurants
a362	*c362*	Alcoholic beverages at limited-service restaurants
a363	*c363*	Alcoholic beverages at all other food and drinking places
a364	*c364*	Mineral procurement for table salt
a365	*c365*	Organic and other chemical procurement for food ingredients
Voucher ID		**Description**
v01		Vouchers for meals at work
v02		Vouchers for employer furnished food and beverages
v03		Vouchers for institutional furnished meals and food assistance
v04		Vouchers for school furnished meals
v05		Vouchers for food and beverage research and development
v06		Beef cattle procurement for retail
v07		Pork animal procurement for retail
v08		Other meat animal procurement for slaughter
v09		Raw milk procurement for fresh milk retail
v10		Raw milk procurement for dairy products except milk retail
v11		Fresh fruit procurement for retail supply
v12		Fresh vegetable procurement for retail supply
v13		Aquaculture procurement for packaging and for final market sales
vo1		Overhead labor cost for redeemed institutional and school vouchers
vo2		Overhead costs for other vouchers (*v01, v02, v05*)

a = activity set element. *c* = commodity set element. *v* = voucher set element. *vo* = voucher overhead set element.

Source: USDA, Economic Research Service

[39] This disaggregation is an intermediate step. In a later section we describe our approach to achieving complete passthrough accounting of each individual caloric commodity marketed for domestic consumption. Passthrough accounting is discussed above in the context of wholesale/retail accounting.

We first created new extended dimension tier 0 accounts and subaccounts with the extended dimensions reflecting new activities and commodities (**rSAM3**). Table 4 lists all the decoupled caloric activities/commodities/vouchers of the expanded Ag-FEDS accounts.

46.a $\mathbf{rSAM3}^{\tilde{\tau}}(F2,F2) = 0$

46.b $\mathbf{rSAM3}^{\tilde{\tau}}(F,F) = \mathbf{rSAM2}^{\tilde{\tau}}(F,F)$

46.c $\mathbf{PCEb3}^{\tilde{\tau}}(F2{\times}O{\times}E) = 0$

46.d $\mathbf{PCEb3}^{\tilde{\tau}}(F{\times}O{\times}E) = \mathrm{vec}[\mathbf{PCEb2}^{\tilde{\tau}}(F{\times}O,FE)]$

Equation 46 maps out the new **rSAM3** and **PCEb3** accounts of dimension *F2* and inserts the entire content of **rSAM2** and **PCEb2** within this larger dimension account, while equation 47 moves the caloric commodities (*cal*) from the *Afs* activities to the *Afsc* activities.

For each foodservice activity (*Afs*), move the caloric commodities (*cal*) to the new caloric procurement activities (*Afsc*) to capture their purchases of caloric commodities, then zero these purchases out from the *Afs* activities:

47.a $\mathbf{rSAM3}^{\tilde{\tau}}(cal,Afsc) = \mathbf{rSAM2}^{\tilde{\tau}}(cal,Afs) \times \mathbf{\Omega}_{Afs,Afsc}$

47.b $\mathbf{rSAM3}^{\tilde{\tau}}(cal,Afs) = 0$

Assembly of the three foodservice commodities from corresponding activities is scaled back by the value of reallocated caloric commodities and these foodservice caloric commodity groups (*fsc*) are assembled from corresponding activities. In doing this, note that each of the three *Afs* activities are assembled into up to all three *fs* commodities so the scaling must be adjusted (**ADJ**) proportional on their prescaling shares:

48.a $\mathbf{ADJ}^{\tilde{\tau}}(Afs,fs) = [\mathbf{\Omega}_{1,cal}{\times}\mathbf{rSAM3}^{\tilde{\tau}}(cal,Afsc){\times}\mathbf{\Omega}_{Afsc,Afs}]'' \times$

48.b $[\{[\mathbf{rSAM3}^{\tilde{\tau}}(Afs,fs){\times}\mathbf{\Omega}_{fs,1}]''\}^{-1}{\times}\mathbf{rSAM3}^{\tilde{\tau}}(Afs,fs)]$

49. $\mathbf{rSAM3}^{\tilde{\tau}}(Afs,fs) = \mathbf{rSAM3}^{\tilde{\tau}}(Afs,fs) - \mathbf{ADJ}^{\tilde{\tau}}(Afs,fs)$

50. $\mathbf{rSAM3}^{\tilde{\tau}}(Afsc,fsc) = \mathbf{\Omega}_{Afsc,fsc} \times (\mathbf{\Omega}_{1,cal} \times \mathbf{rSAM3}^{\tilde{\tau}}(cal,Afsc) \times \mathbf{\Omega}_{Afsc,fsc})''$

With this decoupling and disaggregation of foodservices and their caloric inputs, all activity and institutional purchases of foodservice must also be decoupled and disaggregated. For foodservice activities and institutions, a portion of these outlays are redistributed to the new caloric procurement commodities assembled in equation 50 from output of the activities created in equation 47.a. An important ratio calculation to facilitate these steps is the foodservices share (**fsshr**) of combined foodservice plus caloric commodity outlays by foodservice activity:

51. $\mathbf{fsshr}^{\tilde{\tau}}(fs) = [(\mathbf{\Omega}_{1,Afs}{\times}\mathbf{rSAM3}^{\tilde{\tau}}(Afs,fs)) \times \{(\mathbf{\Omega}_{1,Afs}{\times}\mathbf{rSAM2}^{\tilde{\tau}}(Afs,fs))''\}^{-1}]'$

52. $\mathbf{rSAM3}^{\tilde{\tau}}(fsc,Afs) = \mathbf{\Omega}_{fsc,fs} \times ([1\text{-}\mathbf{fsshr}^{\tilde{\tau}}(fs)]'' \times \mathbf{rSAM3}^{\tilde{\tau}}(fs,Afs))$

53. $\mathbf{rSAM3}^{\tilde{\tau}}(fs,Afs) = \mathbf{fsshr}^{\tilde{\tau}}(fs)'' \times \mathbf{rSAM3}^{\tilde{\tau}}(fs,Afs)$

To obtain the **fsshr** measure in equation 51 we took advantage of the fact that the combined values of *fs* and *fsc* purchased by *Afs* are reflected in the assembly of *fs* in our previous iteration of this account, $\mathbf{rSAM2}^{\tilde{\tau}}$(A*fs*,*fs*), whereas the caloric values have already been netted out in the current iteration (equation 49). The caloric share of total outlays on calories and foodservice equals (1-**fsshr**), and we used this measure to claim a portion of the prescaled values in $\mathbf{rSAM3}^{\tilde{\tau}}(fs,Afs)$ and assign this value in equation 52 to outlays on the *fsc* commodity bundle. Equation 53 scales back the initial value of outlays on *fs* by *Afs* based on **fsshr**. The same procedures apply to institutional buyers:[40]

52h. $\mathbf{rSAM3}^{\tilde{\tau}}(fsc,Fh) = \mathbf{\Omega}_{fsc,fs} \times ([1\text{-}\mathbf{fsshr}^{\tilde{\tau}}(fs)]'' \times \mathbf{rSAM3}^{\tilde{\tau}}(fs,Fh))$

53h. $\mathbf{rSAM3}^{\tilde{\tau}}(fs,Fh) = \mathbf{fsshr}^{\tilde{\tau}}(fs)'' \times \mathbf{rSAM3}^{\tilde{\tau}}(fs,Fh)$

Whereas equations 49–50 establish total supply of the *fs* and *fsc* commodity groups and equations 52–53 assign some uses of these commodity groups, we have not yet assigned uses representing intermediate business expenses across all domestic business activities. This remaining use is addressed after we establish our voucher accounting system. Until then the overall account (**rSAM3**) will be out of balance.[41]

A Voucher Accounting System

Beyond the dollars spent at grocery stores and restaurants, a substantial amount of the food and beverages purchased for domestic consumption have been provided by employers or institutions. Accounting for these acquisitions within the **rSAM** accounting framework means all food and beverage commodities are locked into the fixed technology structure of these accounts. To unlock these expenditures, we introduced a voucher subsystem of accounts and classified these into five categories as follows: (1) meals as a business expense of employers/proprietors (*v01*); (2) food and beverages furnished at work (*v02*); (3) meals at schools and colleges (*v03*); (4) meals provided at other institutions[42] (*v04*); and (5) overhead costs for voucher redemptions (*Vo*). The voucher overhead accounts comprise of two elements, *vo1* and *vo2* (see table 4 for descriptions). Overall acquisitions through these channels represented a market value of several hundred billion dollars in 2017 (USDOC, BEA, 2024c). For example, more than $200 billion in business expensed meals were recorded in 2017.[43] A substantial portion of what USDA, ERS measured to be $133 billion in 2017 for employer and institutionally furnished food and beverages have also been embedded in these accounts. BEA measured $53 billion in their personal consumption expenditure (PCE) accounts for schools, colleges, and furnished food in 2017, which suggests roughly $80 billion in additional employer and institutionally furnished food and beverages have has been embedded in the endogenous transaction accounts. One other category that is a substantial market channel for food and beverage commodities has been those purchased for research and development[44] (*v05*).

It is important to note that our use of the term vouchers has been intended to be an accounting technique to facilitate a separate accounting of transactions that we have worked to reclassify or redirect. For example, when an employer covers employee meal costs while on business travel, we kept track of this food expenditure, whereas economic accounting has, correctly, treated this as a business expense. Although vouchers are not actually provided, this approach allowed us to preserve correct economic accounting and to keep explicit track of food expenditures. For this and other examples discussed below, we introduced this voucher accounting technique. This voucher approach allowed us to avoid double counting when redirecting and/or reclassifying targeted transactions.

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[40] For brevity, we drop the '*F2*' designation for the expanded accounts (table 4) and revert to using '*F*'.

[41] It is important to repeatedly verify overall account balance after every step, being mindful that some steps may temporarily place the accounts out of balance.

[42] These include hospitals, nursing homes, prisons, and both public and private food assistance.

[43] This roughly $200 billion value for purchased meals as a business expense in 2017 accounts for most of the difference between USDA, ERS's measure of total 2017 domestic spending at foodservice establishments and BEA's foodservices PCE.

[44] According to the BEA's 2017 'Use Table, Before Redefinitions, Purchasers' Value' table, the market value of agricultural and manufactured food and beverage commodities purchased by scientific research and development service activities totaled over $3.4 billion. This category (*v05*) does not meet our definition of food and beverage expenditures.

For our voucher subaccounting system, denote $A\mu\lambda$ where $\lambda \in \{v02,v03,v04,v05\}$ the subset of all activities that acquire food and beverage commodities (*fnb*) that will be replaced by acquisitions of type λ vouchers that can be redeemed for *fnb* commodities.

54. $\mathbf{rSAM3}^{\tilde{\tau}}(fnb,\lambda) = \mathbf{rSAM3}^{\tilde{\tau}}(fnb,A\mu\lambda) \times \Omega_{A\mu\lambda,\lambda},\ \forall\ \lambda \in \{v02,v03,v04,v05\}$

55. $\mathbf{rSAM3}^{\tilde{\tau}}(\lambda,A\mu\lambda) = \mathbf{\Omega}_{\lambda,fnb} \times \mathbf{rSAM3}^{\tilde{\tau}}(fnb,A\mu\lambda),\ \forall\ \lambda \in \{v02,v03,v04,v05\}$

56. $\mathbf{rSAM3}^{\tilde{\tau}}(fnb,A\mu\lambda) = 0,\ \forall\ \lambda \in \{v02,v03,v04,v05\}$

57.a $\mathbf{rSAM3}^{\tilde{\tau}}(vo1,A\mu\lambda) = [\mathbf{\Omega}_{vo,\lambda} \times \mathbf{rSAM3}^{\tilde{\tau}}(\lambda,A\mu\lambda)]$

57.b $\times (\mathbf{rSAM3}^{\tilde{\tau}}(Fh,a342) \times \{\mathbf{\Omega}_{1,cal} \times \mathbf{rSAM3}^{\tilde{\tau}}(cal,a359)\}^{-1}),\ \forall\ \lambda \in \{v03,v04\}$

58. $\mathbf{rSAM3}^{\tilde{\tau}}(Fh,A\mu\lambda) = \mathbf{rSAM3}^{\tilde{\tau}}(Fh,A\mu\lambda)$ - $\mathbf{\Omega}_{Fh,vo} \times \mathbf{rSAM3}^{\tilde{\tau}}(vo,A\mu\lambda),\ \forall\ \lambda \in \{v03,v04\}$

59. $\mathbf{rSAM3}^{\tilde{\tau}}(Fh,vo1) = \mathbf{rSAM3}^{\tilde{\tau}}(Fh,vo) + \mathbf{\Omega}_{Fh,vo} \times \mathbf{rSAM3}^{\tilde{\tau}}(vo,A\mu\lambda) \times \mathbf{\Omega}_{A\mu\lambda,vo},\ \forall\ \lambda \in \{v03,v04\}$

The set of activities using this voucher system are such that each activity can only be associated with one of the four voucher programs, *v02–v05*, and equations 54–56 apply to all four programs. In equation 54, each food and beverage acquisition of all participating activities are pooled and moved over to the voucher program column. For each activity participating in the voucher program, the outlays previously dedicated to *fnb* commodities are instead directed to acquisition of vouchers as described in equation 55. After having been reassigned to both the expenditures on, and acquisitions of their corresponding voucher programs, these values are zeroed out in equation 56. For two of the voucher programs (i.e., school meals (*v03*) and institutional foodservice plus food assistance (*v04*)), the participating activities provide food preparation and food services to the populations they serve. Rather than assume that full or limited foodservice operations are jointly produced by these activities, we only assigned a portion of the participating activity labor costs as overhead for administering these two voucher programs. In equation line 57.a, the expression within squared brackets ([ ]) measures total outlays on vouchers for each participating activity and assigns this same value to the overhead costs for voucher redemption row. Equation line 57.b then scales this value based on the ratio of labor costs (*Fh*) to producer values of caloric inputs (*cal*) for the limited-service restaurant activity (*a359*).[45] This reflects the assumption that equation 57 identifies the labor cost per dollar of caloric inputs for all school meal and other institutional foodservices at the same rate as the limited-service restaurants, the only overhead we estimated. This overhead cost is deducted from the total labor costs of the participating activities in equation 58 and added to the labor costs of the overhead cost voucher column in equation 59.

For each of the voucher activities *v02–v05*, we also must move the margin costs of assembling their *fnb* commodities. This required using the margin accounts that are carried forward, as described at the end of the previous subsection. We must first compile margin allocation parameters and initially set all elements to 0—$\mathbf{\Phi}(Ftr,A\mu\lambda) = 0$; $\boldsymbol{\omega}(Fws,A\mu\lambda) = 0$; $\boldsymbol{\omega}(Frt,A\mu\lambda) = 0$. For this purpose, we singled out food-and-beverage wholesalers (*fw*) and other nondurable wholesalers (*ow*) among all wholesalers (*Fws*), and singled out grocery retailers (*fr*) and general retailers (*or*) among all retailers (*Frt*). We proceed as follows:

60.a $\mathbf{\Phi}(Ftr,A\mu\lambda) = [\mathbf{rSAM3}^{\tilde{\tau}}(Ftr,A\mu\lambda) - \mathbf{rSAM1}^{\tilde{\tau}m}(Ftr,A\mu\lambda)] \times$

60.b $\{(\mathbf{\Omega}_{1,Ftr} \times [\mathbf{rSAM3}^{\tilde{\tau}}(Ftr,A\mu\lambda) - \mathbf{rSAM1}^{\tilde{\tau}m}(Ftr,A\mu\lambda)])''\}^{-1}$

61.a $\boldsymbol{\omega}(fw,A\mu\lambda) = \mathbf{rSAM3}^{\tilde{\tau}}(fw,A\mu\lambda) \times \{[\mathbf{\Omega}_{1,Fnbfs} \times \mathbf{rSAM1}^{\tilde{\tau}Sews}(fnbfs,A\mu\lambda)]''\}^{-1}$

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[45] Labor costs remain in the limited-service restaurant activity (*a342*), whereas acquisition of caloric inputs was reassigned to the new caloric acquisitions activity (*a359*) above.

61.b If $\boldsymbol{\omega}(fw,\lambda) > 1$ then $\boldsymbol{\omega}(fw,\lambda) = 1, \forall\ \lambda \in A\mu\lambda$

61.c $\boldsymbol{\omega}(ow,A\mu\lambda) = 1 - \boldsymbol{\omega}(fw,A\mu\lambda)$

62.a $\boldsymbol{\omega}(fr,A\mu\lambda) = \mathbf{rSAM3}^{\tilde{\tau}}(fr,A\mu\lambda) \times \{[\boldsymbol{\Omega}_{1,Fnbfs} \times \mathbf{rSAM1}^{\tilde{\tau}Sert}(fnbfs,A\mu\lambda)]''\}^{-1}$

62.b If $\boldsymbol{\omega}(fr,\lambda) > 1$ then $\boldsymbol{\omega}(fr,\lambda) = 1, \forall\ \lambda \in A\mu\lambda$

62.c $\boldsymbol{\omega}(or,A\mu\lambda) = 1 - \boldsymbol{\omega}(fr,A\mu\lambda)$

Equation 60 measures the mode shares for freight services among each targeted activity ($A\mu\lambda$), by netting out any nonfreight use of transportation services and dividing each mode use by total use of all modes. This parameter applies the overall mode mix of each target activity to their food and beverage acquisitions. Equation 61 determines what share of required wholesale services for the handling of food and beverages can be covered by food-and-beverage wholesale services available for each target activity and applies that share to the total requirement for handling the food and beverages reallocated to voucher activities. If the share is less than 100 percent, the remaining product is handled by other nondurable wholesalers. Because these two types of wholesalers more than cover this need over the entire timeseries, no other contingencies were specified. Equation 62 does for grocery retailers and general retailers what equation 61 does for wholesalers. These two parameters facilitate the movement of margin costs to the voucher activities as follows:

63.a $\mathbf{rSAM3}^{\tilde{\tau}}(Ftr,\lambda) = (\boldsymbol{\Phi}(Ftr,A\mu\lambda) \times \boldsymbol{\Omega}_{A\mu\lambda,fnb\times A\mu\lambda}) \times \text{vec}[\mathbf{rSAM1}^{\tilde{\tau}Setr}(fnb,A\mu\lambda)]''$

63.b $\times\ \boldsymbol{\Omega}_{fnb\times A\mu\lambda,\lambda}, \forall\ \lambda \in \{v02,v03,v04,v05\}$

64.a $\mathbf{rSAM3}^{\tilde{\tau}}(F\lambda 1,\lambda 2) = \boldsymbol{\omega}(F\lambda 1,A\mu\lambda 2) \times \boldsymbol{\Omega}_{A\mu\lambda 2,fnb\times A\mu\lambda 2}$

64.b $\times\ (\text{vec}[\mathbf{rSAM1}^{\tilde{\tau}Se\lambda 1}(fnb,A\mu\lambda 2)]'' \times \boldsymbol{\Omega}_{fnb\times A\mu\lambda 2,\lambda 2}), \lambda 1 \in (ws,rt), \lambda 2 \in \{v02,v03,v04,v05\}$

65.a $\mathbf{rSAM3}^{\tilde{\tau}}(F\lambda 1,A\mu\lambda 2) = \mathbf{rSAM3}^{\tilde{\tau}}(F\lambda 1,A\mu\lambda 2) + \boldsymbol{\Omega}_{F\lambda 1,fnb\times A\mu\lambda 2}$

65.b $\times\ (\text{vec}[\mathbf{rSAM1}^{\tilde{\tau}Se\lambda 1}(fnb,A\mu\lambda 2)]'' \times \boldsymbol{\Omega}_{fnb\times A\mu\lambda 2,A\mu\lambda 2}), \lambda 1 \in (ws,rt), \lambda 2 \in \{v02,v03,v04,v05\}$

66.a $\mathbf{rSAM3}^{\tilde{\tau}}(Ftr,A\mu\lambda) = \mathbf{rSAM3}^{\tilde{\tau}}(Ftr,A\mu\lambda) + \boldsymbol{\Omega}_{Ftr,fnb\times A\mu\lambda}$

66.b $\times\ (\text{vec}[\mathbf{rSAM1}^{\tilde{\tau}Setr}(fnb,A\mu\lambda)]'' \times \boldsymbol{\Omega}_{fnb\times A\mu\lambda,A\mu\lambda}), \lambda \in \{v02,v03,v04,v05\}$

67.a $\mathbf{rSAM3}^{\tilde{\tau}}(Ftr,A\mu\lambda) = \mathbf{rSAM3}^{\tilde{\tau}}(Ftr,A\mu\lambda) - \boldsymbol{\Phi}(Ftr,A\mu\lambda) \times \boldsymbol{\Omega}_{A\mu\lambda,fnb\times A\mu\lambda}$

67.b $\times\ (\text{vec}[\mathbf{rSAM1}^{\tilde{\tau}Setr}(fnb,A\mu\lambda)]'' \times \boldsymbol{\Omega}_{fnb\times A\mu\lambda,A\mu\lambda}), \lambda \in \{v02,v03,v04,v05\}$

68.a $\mathbf{rSAM3}^{\tilde{\tau}}(F\lambda 1,A\mu\lambda 2) = \mathbf{rSAM3}^{\tilde{\tau}}(F\lambda 1,A\mu\lambda 2) - \boldsymbol{\omega}(F\lambda 1,A\mu\lambda 2) \times \boldsymbol{\Omega}_{A\mu\lambda 2,fnb\times A\mu\lambda 2}$

68.b $\times\ (\text{vec}[\mathbf{rSAM1}^{\tilde{\tau}Se\lambda 1}(fnb,A\mu\lambda 2)]'' \times \boldsymbol{\Omega}_{fnb\times A\mu\lambda 2,A\mu\lambda 2}), \lambda 1 \in (ws,rt), \lambda 2 \in \{v02,v03,v04,v05\}$

Equations 63 and 64 move all freight and trade margin costs linked to the food and beverages now assembled by voucher accounts to those same accounts using the freight mode, wholesale, and retail service mixes described in the $\boldsymbol{\Phi}(Ftr,A\mu\lambda)$, $\boldsymbol{\omega}(Fws,A\mu\lambda)$, and $\boldsymbol{\omega}(Frt,A\mu\lambda)$ parameters. Equations 65 and 66 convert these outsourced margin costs among the target activities to additional voucher acquisitions, whereas equations 67 and 68 remove the costs of these margin outlays that were outsourced to avoid double counting.

To establish the voucher account for meals at work (*v01*), we denote *Axfs* the set of all nonfoodservice activities that acquire foodservice commodities (*fs*) that will be replaced by acquisitions of vouchers that can be redeemed to acquire foodservices and accompanying caloric commodities (*cal*):

69. $\mathbf{rSAM3}^{\tilde{\tau}}(fs,v01) = \mathbf{rSAM3}^{\tilde{\tau}}(fs,Axfs) \times \Omega_{Axfs,v01}$

70. $\mathbf{rSAM3}^{\tilde{\tau}}(v01,Axfs) = \boldsymbol{\Omega}_{v1,fs} \times \mathbf{rSAM3}^{\tilde{\tau}}(fs,Axfs)$

71. $\mathbf{rSAM3}^{\tilde{\tau}}(fs,Axfs) = 0$

Equations 69–71 involve the reallocation of foodservice outlays to vouchers for meals at work among all nonfoodservice activities that have meals at work expenses. With all intermediate outlays on foodservices now consolidated in the foodservices voucher group (*v01*), we can complete the splitting of these outlays in the same manner as was done above for the foodservice activities and households:

72. $\mathbf{rSAM3}^{\tilde{\tau}}(fsc,v01) = \boldsymbol{\Omega}_{fsc,fs} \times [1\text{-}\mathbf{fsshr}^{\tilde{\tau}}(fs)]'' \times \mathbf{rSAM3}^{\tilde{\tau}}(fs,v01)]$

73. $\mathbf{rSAM3}^{\tilde{\tau}}(fs,v01) = \mathbf{fsshr}^{\tilde{\tau}}(fs)'' \times \mathbf{rSAM3}^{\tilde{\tau}}(fs,v01)$

Finetuning to Enhance Performance of Multipliers

Any negative intermediate outlays by activities or negative import values for margin industries are addressed; redefinitions of targeted nonfood wholesale and retail services address deficits in food wholesale (*fw*) and food retail (*fr*) services that facilitate all final market food expenditures; and **rSAM** and **PCEb** accounts are combined into a single expanded **rSAM** account:

- Equations 72 converts negative commodity outlays by activity into secondary activity outputs, and equation 73 redefines negative transportation and wholesale imports into exports.
- When BEA's reported wholesale and retail services facilitating final demand does not have sufficient *fw* and *fr* services, equation 74 makes up this deficit by redefining other nondurable wholesales as *fw* and general merchandise retailers as *fr* to ensure that all trade services facilitating final market food and beverage sales are exclusively by *fw* and *fr* services.
- Disaggregation of the institutional final demand accounts in equation 75 begins by deducting from personal consumption expenditures (*Xh*), all food and beverage related expenditures (*Xf*) defined within the **PCEb** account include:
  - Equation 75.a inserts all food, beverage, and foodservice commodity purchases for the *Xf* columns (table 5) of **rSAM3**, as measured in producer prices within the relevant expenditure (*E*) categories of **PCEb3**.
  - Equation 75.b inserts all food wholesale (*fw*) and food retail (*fr*) commodity purchases for the *Xf* columns of **rSAM3**, as measured in wholesale/retail margins within relevant expenditure (*E*) categories of **PCEb3** and since we know there is no deficit of *fw* and *fr*, these values are assigned to *fw* and *fr* rows.
  - Equation 75.c performs the same calculation as 75.b for transportation costs, splitting costs to specific freight modes based on the known (**rSAM3**) combined mode cost shares of the full and limited foodservice activities.
  - We deduct assigned costs in 75.a to 75.c from **rSAM3** *Xh* column, which is redefined as nonfood PCE.

Researchers can routinely apply several finetuning techniques to better align multiplier model calculations for their intended uses. These techniques include: (1) address negative value entries (with a few notable exceptions) by reversing direction of flows; (2) measure and address deficits in food wholesale and retail services that facilitate all food dollar expenditures; and (3) integrate the two tier 0 accounts (**rSAM** and **PCEb**) into a single expanded **rSAM** account to significantly increase the number of commodity and marketing channel food dollar statistical series that can be annually reported.

Addressing Negative Value Entries

It is not unusual for source data within both the BEA's annual and benchmark year use tables published to include some negative value entries. Common examples include the representation of secondhand goods production as a byproduct of an activity, or the representation of domestic wholesale services employed to facilitate the import of commodities as a negative import of these services. With a few exceptions, such as the measure of government subsidies paid to specific activities, these negative values can lead to misleading statistics. For example, the case where household expenditures on coffee, tea, and other beverage materials has produced recycling materials as a $1.2-billion byproduct and this gets recorded as a negative expenditure on scrap under the coffee, tea, and other beverage PCE account ("PCE Bridge Table, 2017" from USDOC, BEA, 2024d). Although this is a small percentage of the $19.3 billion in annual coffee, tea, and other beverage spending, it does lead to an understatement of consumer spending on coffee, tea, and related drinks. Our remedy has been to redirect flows using one of two approaches depending on the category of transaction:

72. If $\mathbf{rSAM3}^{\tilde{\tau}}(\lambda 1,\lambda 2) < 0$: $\mathbf{rSAM3}^{\tilde{\tau}}(\lambda 2,\lambda 1) = -\mathbf{rSAM3}^{\tilde{\tau}}(\lambda 1,\lambda 2)$; $\mathbf{rSAM3}^{\tilde{\tau}}(\lambda 1,\lambda 2) = 0, \forall \lambda 1 \in FC, \lambda 2 \in FA$

73. If $\mathbf{rSAM3}^{\tilde{\tau}}(\lambda 1,\lambda 2) < 0$: $\mathbf{rSAM3}^{\tilde{\tau}}(\lambda 2,\lambda 1) = -\mathbf{rSAM3}^{\tilde{\tau}}(\lambda 1,\lambda 2)$; $\mathbf{rSAM3}^{\tilde{\tau}}(\lambda 1,\lambda 2) = 0, \forall \lambda 1 \in Fr2, \lambda 2 \in Fr1$

In equation 72, any negative outlay on commodity $\lambda 1$ by activity $\lambda 2$ is converted to joint output of commodity $\lambda 1$ by activity $\lambda 2$, such that both supply and use of the commodity increase by the absolute value of the negative outlay and the account remains balanced. In equation 73, any negative import value reported for commodity $\lambda 1$ is converted to a commodity export value of $\lambda 1$ such that both imports and exports of the commodity increase by the absolute value of the negative import, and GDP is unchanged while the account remains balanced.

Address Deficits in Food Wholesale and Retail Services

Another finetuning step we addressed was to single out wholesalers and retailers handling food and beverage commodity transactions between farmers plus food processors on the supply side, and household plus foodservice/institutional buyers who prepare and consume or serve these commodities on the demand side. We denoted these activities and commodities as *Afwfr* and *fwfr,* respectively. These entities specialize in food/beverage wholesaling (*fw*) and retailing (*fr*) services, which are different than the wholesalers who facilitate other sales such as the purchase of farm and processing machinery, or the office supply retailer who facilitates the purchase of office supplies to businesses providing business services to support the food economy. For example, food and beverages merchants are part of the cold chain, which always keeps perishable products within refrigerators and freezers requiring greater amounts of electricity. In supply chain analysis using Ag-FEDS, we included the *Afwfr* wholesalers and retailers in supply chain industry groups and included the other types of wholesalers/retailers in our subcontracting industry group.

Recognizing that some food transactions may be facilitated by wholesalers and retailers that are not classified as *Afwr*, when they are used for such purposes, we redefined them as food wholesalers/retailers.[46] This had only been necessary when our source data in the **rSAM** and **PCEb** accounts indicated a deficit of *fwfr* services sold to facilitate personal consumption expenditures. To address such deficits when they exist, we redefined a portion of available other nondurable wholesaler and general merchandise retailer activities (*Aowor*) and commodities (*owor*) to cover any deficits to facilitating all target food and beverage sales as follows:

[46] The alternative is to let all food and beverage transactions draw from the pool of all wholesalers and retailers which would greatly understate the energy inputs that uniquely define the *Afwfr* technologies.

Table 5
Agri-food economic data system's (Ag-FEDS) food, beverage, and food service expenditure tables

Column	Description
Xf0000	**Food and food-related dollar**
Xf1000	**Food dollar**
Xf1100	**Food-at-home dollar**
Xf1101	Food at home: Cereals
Xf1102	Food at home: Bakery products
Xf1103	Food at home: Beef
Xf1104	Food at home: Pork
Xf1105	Food at home: Other meats
Xf1106	Food at home: Poultry
Xf1107	Food at home: Fish and seafood
Xf1108	Food at home: Fresh milk
Xf1109	Food at home: Processed dairy products
Xf1110	Food at home: Fresh eggs
Xf1111	Food at home: Processed eggs
Xf1112	Food at home: Fats and oils (including mayonnaise)
Xf1113	Food at home: Fresh fruits
Xf1114	Food at home: Fresh vegetables
Xf1115	Food at home: Canned, frozen, and dried fruits and vegetables
Xf1116	Food at home: Sugar and sweets
Xf1117	Food at home: Snack foods
Xf1118	Food at home: Frozen prepared foods
Xf1119	Food at home: Processed fruit and vegetable canning and drying (e.g., soups, catsup, pickles)
Xf1120	Food at home: Seasonings, sauces (except tomato), and dressings (excluding mayonnaise)
Xf1121	Food at home: Dry, condensed, and evaporated nondairy products
Xf1122	Food at home: Tree nuts and peanuts (unprocessed)
Xf1123	Food at home: Fresh cut produce plus grab and go foods
Xf1124	Food at home: Miscellaneous foods and ingredients
Xf1125	Food at home: Fruit and vegetable juices
Xf1126	Food at home: Food consumed on farms
Xf1127	Beverage at home: Coffee, tea, and beverage materials (except soft drinks)
Xf1128	Beverage at home: Soft drinks and bottled water
Xf1200	**Food-away-from-home dollar**
Xf1201	Food away: Meals and nonalcoholic beverages at full-service eating places
Xf1202	Food away: Meals and nonalcoholic beverages at limited-service eating places
Xf1203	Food away: Meals and nonalcoholic beverages at other food and drinking places
Xf1204	Food away: School meals and nonalcoholic beverages
Xf1205	Food away: Food and nonalcoholic beverages furnished to employees (including military)
*Xf1206**	Institutional furnished food and nonalcoholic beverages and food assistance
*Xf1207**	Food and nonalcoholic beverages at work (per diem and expensing)

continued on next page ►

Table 5 (cont.)

Agri-food economic data system's (Ag-FEDS) food, beverage, and food service expenditure tables

Column	Description
Xf2000	**Alcoholic beverage dollar**
Xf2100	**Alcoholic beverage-at-home dollar**
Xf2101	Beverage at home: Beer
Xf2102	Beverage at home: Wine
Xf2103	Beverage at home: Spirits
Xf2200	**Alcoholic beverage-away-from-home dollar**
Xf2201	Beverages away: Alcohol at full-service eating places
Xf2202	Beverages away: Alcohol at limited-service eating places
Xf2203	Beverages away: Alcohol at other food and drinking places

Xf = all food and beverage related consumption expenditures that have been separated from other consumption expenditures measured in account *Xh*.

Note: Tables *Xf1206* and *Xf1207* are redefined food services previously categorized as either an intermediate production costs (e.g., paying for employee meals during business travel) or as a part of a group of institutional services (e.g., school meals as part of education services by local government).

Source: USDA, Economic Research Service

74.a $\text{deficit}_{\lambda 1||\lambda 2} = \mathbf{rSAM3}^{\tilde{\tau}}(\lambda 2,Fh) - \mathbf{\Omega}_{1,FC\times\lambda 1\times Xfbah}\times\mathbf{PCEb3}^{\tilde{\tau}}(FC\times\lambda 1\times fbah)$

If $\text{deficit}_{\lambda 1||\lambda 2} < 0$:

74.b $\mathbf{rSAM3}^{\tilde{\tau}}(A\lambda 3,\lambda 2) = \mathbf{rSAM3}^{\tilde{\tau}}(A\lambda 3,\lambda 2) - \text{deficit}_{\lambda 1||\lambda 2}$

74.c $\mathbf{rSAM3}^{\tilde{\tau}}(A\lambda 3,\lambda 3) = \mathbf{rSAM3}^{\tilde{\tau}}(A\lambda 3,\lambda 3) + \text{deficit}_{\lambda 1||\lambda 2}$

74.d $\mathbf{rSAM3}^{\tilde{\tau}}(\lambda 3,Fh) = \mathbf{rSAM3}^{\tilde{\tau}}(\lambda 3,Fh) + \text{deficit}_{\lambda 1||\lambda 2}$

74.e $\mathbf{rSAM3}^{\tilde{\tau}}(\lambda 2,Fh) = \mathbf{rSAM3}^{\tilde{\tau}}(\lambda 2,Fh) - \text{deficit}_{\lambda 1||\lambda 2}$,

74.f $\lambda 1||\lambda 2||\lambda 3 \in \{wsfwow, rtfror\}$

Equation 74 ensures that available *fw* and *fr* services in the household subaccount ($\mathbf{rSAM3}^{\tilde{\tau}}(fwfr,Fh)$) are sufficient to meet the total wholesale (*ws*) and retail (*rt*) requirements reported in the PCE bridge table ($\mathbf{PCEb3}(FC\times wsrt\times fbah)$) under the off premises food consumption category and denoted *fbah* (food and beverages at home). Equation 74.a measures this and a deficit is indicated by a negative value for this equation. Equations 74.b–74.e are conditional on there being a deficit for $\lambda 1||\lambda 2 = wsfw$ and/or *rtfr*. In 74.b a portion of the *Aow* (other nondurable wholesalers) and/or *Aor* (general merchandise retailers) activity output is redefined as joint production of the *fw* and/or *fr* commodity by an amount equal to the absolute value of the measured deficit, whereas 74.c reduces the quantity of *ow* and/or *or* commodity production by the same amount from these same activities. In 74.d this newly available *fw* and/or *fr* commodity production is embedded in the household subaccount (*Fh*) to cover any measured deficits, and the scaled back *ow* and/or *or* commodity production is achieved by its reduced use in the household subaccount as measured in 74.e. Equation 74.f ensures that only the ordered triplets, $\lambda 1||\lambda 2||\lambda 3=\{wsfwow\}$ and $\lambda 1||\lambda 2||\lambda 3=\{rtfror\}$, are considered.

*Integrating Tier 0 Accounts into a Single Expanded **rSAM** Account*

Our last task in this section is to integrate the two tier 0 accounts (**rSAM** and **PCEb**) into a single expanded **rSAM** account to significantly expand the number of commodity and marketing channel food dollar statistical series tables that can be annually reported. For each detailed food and beverage (*fbah*) or foodservice (*fbafh*, food and beverages away from home) expenditure (*E*) category, sometimes we further broke out one or more specific commodity expenditures within each category to develop a new column within the injection matrix (**X**) subaccount (figure 5). These new columns are deducted from the appropriate institutional column from where they are extracted. Table 5 lists these new columns along with their hierarchical subtotals, each of which become a new food dollar table. They are compiled from the **PCEb** account and incorporated into the **rSAM** account as follows:

75.a $\mathbf{rSAM3}^{\tilde{\tau}}(FC,Xf) = \mathbf{\Omega}_{FC,FC\times p\times E} \times (\mathbf{PCEb3}^{\tilde{\tau}}(FC\times p\times E)'' \times \mathbf{\Omega}_{FC\times p\times E,Xf})$

75.b $\mathbf{rSAM3}^{\tilde{\tau}}(fwfr,Xf) = \mathbf{\Omega}_{fwfr,FC\times wsrt\times E} \times (\mathbf{PCEb3}^{\tilde{\tau}}(FC\times wsrt\times E)'' \times \mathbf{\Omega}_{FC\times wsrt\times E,Xf})$

75.c $\mathbf{rSAM3}^{\tilde{\tau}}(trn,Xf) = \mathbf{shrtrn}(trn) \times \mathbf{\Omega}_{tr,FC\times tr\times E} \times (\mathbf{PCEb3}^{\tilde{\tau}}(FC\times tr\times E)'' \times \mathbf{\Omega}_{FC\times tr\times E,Xf})$

75.d $\mathbf{rSAM3}^{\tilde{\tau}}(FC,Fh) = \mathbf{rSAM3}^{\tilde{\tau}}(FC,Fh) - \mathbf{rSAM3}^{\tilde{\tau}}(FC,Xf) \times \mathbf{\Omega}_{Xf,Fh}$

Equation 75.a moves all the appropriate *C–E* pairs recorded in producer prices to their appropriate *Xf* columns while zeroing out all other data in the **PCEb** account. In equation 75.b, we summed all wholesale and retail margin costs (*wsrt*) across all transactions per *Xf* column and place these values in the appropriate *fwfr* rows. Then, we can reclassify these generic trade margin costs as food wholesale and retail services because directly above we made sure there was no deficit of these services available in the household institutional account. In equation 75.c, we summed all transportation costs (*tr*) across all transactions per *Xf* column and distributed these values to the appropriate freight transportation rows (*trn*) using a **shrtrn**(*trn*) vector. To conserve on notation, we describe the calculation of this vector. It represents the share of total freight service outlays on each transportation mode (i.e., truck, air, rail, and/or water) combined for the full-service and limited-service activities reported in the **rSAM** account. Our justification is that these foodservice activities generally colocate where households reside. This colocation suggest they acquire food and beverages from the same sources as the food retailers serving these households so should have a similar freight services profile. Having moved all relevant data from the **PCEb** account to the new columns of the **rSAM** account, equation 75.d deducts this entire value by row from the institutional household column. Since we have integrated these two annual tier 0 accounts we were assured that this deduction has not created deficits in the scaled back food and beverage commodity rows. We do not expect (and never encountered) any deficits for any of the freight service rows.

Figure 5

Agri-food economic data system (Ag-FEDS) social accounting matrix (SAM) schematic with voucher subaccounts

Row	Activities (*FA*)	Commodities (*FC*)	Vouchers (*FV*)		Institutions and rest of world		Total
	Fa001 … *Fa363*	*Fc001* … *Fc363*	*Fv01* … *Fv13*	*Fvo1* *Fvo2*	*FXf* *FXh* *FXg* *FXk*	*FXr1*	
Activities *Fa001* ⋮ *Fa363*	**Z**[*FA*,*FA*](=0)	**Z**[*FA*,*FC*] (=**S**[*FA*,*FC*])	**Z**[*FA*,*FVc*](=0)	**Z**[*FA*,*FVo*](=0)	**X**[*FA*,*FXI*](=0)	**X**[*FA*,*FXr1*](=0)	Gross output (**y**)
	Endogenous transactions (ET): inner-inner matrix (Z$_{in,in}$)		**ET: inner-outer matrix (Z$_{in,out}$)**		**Injection matrix (X)**		
Commodities *Fc001* ⋮ *Fc363*	**Z**[*FC*,*FA*] (=**U**[*FC*,*FA*])	**Z**[*FC*,*FC*](=0)	**Z**[*FC*,*FVc*]	**Z**[*FC*,*FVo*](=0)	**X**[*FC*,*FXI*] (=**U**[*FC*,*FXI*])	**X**[*FC*,*FXr1*] (=**U**[*FC*,*FXr1*])	
	ET: outer-inner matrix (Z$_{out,in}$)		**ET: outer-outer matrix (Z$_{out,out}$)**				
Vouchers *Fv01* ⋮ *Fv13*	**Z**[*FVc*,*FA*]	**Z**[*FVc*,*FC*](=0)	**Z**[*FVc*,*FVc*](=0)	**Z**[*FVc*,*FVo*](=0)	**X**[*FVc*,*FXI*]	**X**[*FVc*,*FXr1*](=0)	
Fvo1 *Fvo2*	**Z**[*FVo*,*FA*]	**Z**[*FVo*,*FC*](=0)	**Z**[*FVo*,*FVc*](=0)	**Z**[*FVo*,*FVo*](=0)	**X**[*FVo*,*FXI*](=0)	**X**[*FVo*,*FXr1*](=0)	
Institutions and rest of world *FLh*	**L**[*FLI*,*FA*] (=**U**[*FLI*,*FA*])	**L**[*FLI*,*FC*] (= 0)	**L**[*FLI*,*FVc*]	**L**[*FLI*,*FVo*]	**LX**[*FLI*,*FXI*]	**LX**[*FLI*,*FXr1*](=0)	GDI + imports
FLg							
FLk	**Leakage matrix (L)**				**Exogenous transactions (LX)**		
FLr2	**L**[*FLr2*,*FA*] (= 0)	**L**[*FLr2*,*FC*] (=(-**U**[*FC*,*FLr2*])')	**L**[*FLr2*,*FVc*](=0)	**L**[*FLr2*,*FVo*]	**LX**[*FLr2*,*FXI*](=0)	**LX**[*FLR*,*FLR*](=0)	
Total	Gross output (**y'**)				GDP + imports		

Z = Endogenous transactions. **X** = Injection matrix. **L** = Leakage matrix. **LX** = Exogenous transactions. GDP = Gross Domestic Product. GDI = Gross domestic income. *FA* = activity subset of *F* account. *FC* = commodity subset of *F* account. *FXI* = Institutional subset of *F* account within injection matrix. *FLI* = Institutional subset of *F* account within leakage matrix. *FXr1* = Rest-of-world exports subset of *F* account within injection matrix. *FXr2* = Rest-of-world imports subset of *F* account within leakage matrix. *FVc* = is the voucher commodity subset of *F* account. *FVo* = is the voucher overhead subset of *F* account.

Note: Institutional accounts (*I*) are partitioned into households (*h*), governments (*g*), and saving/investment (*k*). Rest-of-world accounts (*R*) are partitioned into international exports (*r1*), and international imports (*r2*).

Source: USDA, Economic Research Service adapted from Canning, P., Rehkamp, S., & Yi, J. (2022). Environmental input-output models for food systems research: Application and extensions. In C. J. Peters & D. D. Thilmany (Eds.), *Food systems modelling: Tools for assessing sustainability in food and agriculture* (pp. 179–211). Elsevier.

3.3 Measuring the Hidden Food Economy

Several USDA, ERS annual data products have used a consistent approach and, in some cases, the same data sources as the BEA system of national accounts, but with more granularity with respect to the agri-food economy. Two examples that directly relate to our Ag-FEDS accounts are USDA, ERS' Food Expenditure Series (FES) (USDA, ERS, 2024a) and Price Spreads from Farm to Consumer Series (PSF2C) (USDA, ERS, 2024b). The information contained in these two series get obscured in the national accounts, largely due to accounting conventions and data aggregation.

Reconciling the USDA, Economic Research Service's Food Expenditure Series and U.S. Department of Commerce, Bureau of Economic Analysis' Personal Consumption Expenditures

We have identified several sources of discrepancies between the USDA, ERS Food Expenditure Series (FES) and the U.S. Department of Commerce, Bureau of Economic Analysis's (BEA) Personal Consumption Expenditures (PCE) and address these by broad expenditure categories:

- Equations 77–80 concern total food and alcohol at home expenditures:
  - BEA overcounting both total food (FAH) and alcohol (AAH) at home expenditures as measured by USDA, ERS is largely attributed to the passthrough accounting of retail expenditures and equation 77 scales all food at home (*Xffah*) and alcohol at home (*Xfaah*) columns down by this measured discrepancy in the intermediate **rSAM4** (table 6).
  - BEA undercounting both total food (FAFH) and alcohol (AAFH) at foodservice establishments as measured by USDA, ERS is also partly attributed to passthrough accounting. All surplus at-home spending deducted in equation 77, or the share of it if it exceeds the away-from-home deficits, is distributed to the three foodservice marketing channels in proportion to each of their market share of total foodservices. The total at home surpluses attributed to the *Xffafh* and *Xfaafh* accounts is measured in equation 79.
  - If either *Xffah* or *Xfaah* surplus exceeds their corresponding away-from-home deficits (typically for alcohol and unobserved thus far for food), equation 80 allocates residuals to the *Xh* account in **rSAM4**.
  - All remaining deficits in BEA *Xffah* spending is accounted for in equation 81 by assigning that portion of *v01* vouchers (food and beverages at work) required to cover the remaining deficit to the *Xf1207* account in **rSAM4**. Equation 82 deducts these moved vouchers from the *v01* account. Equation 83 replaces these deducted outlays as an operating surplus of the *v01* account. This added operating surplus is offset by a deduction in operating surplus to the *Xf1207* account in equation 84 (an accounting technique to avoid redefining GDI and GDP).
  - Only a small share of expenditures on school meals and institutionally furnished food, as measured by ERS (USDA, ERS, 2024a), is reflected in the **PCEb** account. Additional spending on these categories was extracted from government and institutional activities and allocated to vouchers *v03* (institutionally furnished) and *v04* (school meals). Identical approaches as described above to allocate *v01* vouchers is applied for *v04* vouchers in equations 85–88, and for *v03* vouchers in equations 89–92.

- Figure 5 captures to new layout of **rSAM4** and the steps of this subsection largely involve movements of outlays between the voucher accounts and the *Xf* accounts depicted in the columns and rows of the figure. The discrepancies between the USDA, ERS Food Expenditure Series and the BEA's Personal Consumption Expenditures in measuring expenditures on food services (*FAFH*) and food for off premises consumption (*FAH*) are summarized in table 6. This section addresses several sources of these discrepancies.

We discussed the passthrough accounting procedures applied to retail activities and highlighted examples that alone account for over 60 percent of the 2017 discrepancy reported in table 6 for food at home and we expect a similar percentage for all years. In the previous section, we also identified more than $100 billion annually in foodservice as an intermediate expense of production activities, representing business related expensing of meals. From a national income accounting perspective, these costs have contributed to the GDP linked to the commodities these businesses produce. USDA, ERS measures these costs as food expenditures and this accounting practice, along with the passthrough accounting already discussed, appears to fully explain the significant discrepancy in foodservice expenditure measures by BEA and USDA, ERS. Other definitional differences between USDA, ERS' and BEA's measures of various institutional activities involving food and foodservices appeared to be fully explained by the food, beverages, and foodservices that we have decoupled and redirected to numerous voucher activities (table 4). In this section, we leveraged this work described in the previous section to reconcile and fully account for every discrepancy reported in table 6. We started by declaring our fourth iteration of the target **rSAM** account:

76. $\mathbf{rSAM4}^{\tilde{\tau}}(F,F) = \mathbf{rSAM3}^{\tilde{\tau}}(F,F)$

Table 6

Comparison of USDA, Economic Research Service's Food Expenditures Series and U.S. Department of Commerce, Bureau of Economic Analysis' Food Personal Consumption Expenditures by Marketing Channel, 1997–2023

Year	Total food at home (FAH)		Foodservice (FAFH)		Meals at schools and colleges (SM)		Institutions and furnished (OIF)		Total alcohol at home (AAH)		Total alcohol away from home (AAFH)		Total	
	ERS	BEA	ERS	BEA	ERS	BEA	ERS	BEA	ERS	BEA	ERS	BEA	ERS	BEA
	Million dollars													
1997	376,328	413,178	275,039	245,328	27,754	12,823	33,602	8,211	51,010	61,598	41,412	32,170	805,146	773,308
1998	384,394	422,469	289,675	257,638	29,258	13,510	34,970	8,423	52,536	64,968	44,439	35,048	835,272	802,056
1999	405,809	445,295	304,206	269,143	31,081	14,497	36,835	8,705	54,530	70,235	47,691	38,181	880,151	846,056
2000	423,770	463,443	329,148	287,419	31,939	15,209	37,581	8,928	57,708	77,136	51,941	42,267	932,086	894,402
2001	443,473	482,514	341,300	297,468	33,924	15,584	38,417	9,135	58,739	81,489	54,441	45,381	970,293	931,571
2002	451,862	490,678	356,992	308,353	36,496	15,918	38,883	9,470	58,420	84,374	58,212	49,318	1,000,866	958,111
2003	471,018	513,853	376,669	325,630	38,591	16,590	40,721	10,226	60,262	85,728	60,093	52,143	1,047,354	1,004,170
2004	496,639	543,335	404,105	348,336	40,951	17,011	43,077	10,876	64,113	89,269	62,115	55,836	1,110,999	1,064,663
2005	526,061	575,666	428,368	369,137	43,598	17,443	45,579	12,135	67,817	92,551	63,784	59,392	1,175,207	1,126,324
2006	551,062	602,029	455,295	390,904	45,767	17,962	49,059	13,922	72,065	98,231	66,827	63,468	1,240,075	1,186,516
2007	579,264	635,116	479,749	409,031	48,439	18,913	52,608	14,727	75,764	102,215	69,017	66,924	1,304,842	1,246,926
2008	605,436	666,209	492,701	419,312	51,788	20,609	54,523	15,014	80,011	102,877	71,109	68,951	1,355,568	1,292,972
2009	594,686	669,496	484,658	414,516	56,798	21,736	54,858	15,597	78,760	103,434	70,688	68,170	1,340,448	1,292,949
2010	610,421	678,972	501,557	427,656	59,816	23,364	53,416	15,305	82,955	107,895	73,508	70,761	1,381,673	1,323,953
2011	642,246	709,419	534,610	448,221	61,832	24,206	56,299	16,779	88,252	110,123	78,924	74,703	1,462,163	1,383,451

continued on next page ►

Table 6 (cont.)

Comparison of USDA, Economic Research Service's Food Expenditures Series and U.S. Department of Commerce, Bureau of Economic Analysis' Food Personal Consumption Expenditures by Marketing Channel, 1997–2023

Year	Total food at home (FAH)		Foodservice (FAFH)		Meals at schools and colleges (SM)		Institutions and furnished (OIF)		Total alcohol at home (AAH)		Total alcohol away from home (AAFH)		Total	
	ERS	BEA	ERS	BEA	ERS	BEA	ERS	BEA	ERS	BEA	ERS	BEA	ERS	BEA
	Million dollars													
2012	684,129	732,026	568,802	470,547	63,439	24,551	57,680	17,134	93,487	114,172	85,605	79,252	1,553,141	1,437,682
2013	697,718	753,166	607,614	488,888	63,654	24,984	59,007	17,866	98,410	117,369	87,332	81,850	1,613,734	1,484,123
2014	723,845	785,923	659,660	517,590	62,774	26,048	61,264	19,054	104,629	124,498	92,208	85,904	1,704,380	1,559,017
2015	733,226	809,560	722,995	556,313	65,103	26,923	64,400	20,836	108,157	132,446	98,633	91,632	1,792,514	1,637,710
2016	744,362	829,436	772,196	583,040	65,915	27,475	68,272	23,134	114,328	140,161	103,831	95,339	1,868,905	1,698,585
2017	771,721	862,988	828,040	611,961	65,657	27,699	70,983	24,701	122,672	147,456	109,078	99,191	1,968,151	1,773,996
2018	799,481	889,542	873,602	645,479	66,774	27,698	73,385	25,857	129,367	154,838	115,061	104,856	2,057,670	1,848,270
2019	824,852	920,825	917,872	679,807	69,518	27,661	75,931	26,632	133,790	162,413	120,924	110,601	2,142,887	1,927,939
2020	886,883	1,010,466	797,332	605,823	65,468	14,822	68,867	23,816	147,132	188,406	86,908	91,573	2,052,589	1,934,906
2021	955,478	1,088,682	1,017,725	763,592	63,812	15,283	79,551	25,672	162,495	203,191	126,830	119,030	2,405,890	2,215,450
2022	1,044,179	1,181,848	1,175,115	870,624	77,721	23,064	82,878	29,535	175,235	213,952	150,701	139,075	2,705,829	2,458,098
2023	1,068,291	1,222,204	1,293,001	956,290	119,828	28,405	90,647	32,531	177,873	221,807	169,545	154,790	2,919,184	2,616,027

Source: USDA, Economic Research Service (ERS) using USDA, ERS Food Expenditure Series data; and U.S. Department of Commerce, Bureau of Economic Analysis (BEA) food personal consumption expenditures data. Data are subject to revisions.

Total Food at Home (FAH) and Total Alcohol at Home (AAH)

Beginning with the two consistently higher BEA expenditure categories (i.e., total food at home and total alcohol at home), we denoted USDA, ERS values as FAH and AAH, respectively, and corresponding *Xf* expenditure categories in the *F* accounts identified in table 5[47] (*Xf1101–Xf1128* and *Xf2101–Xf2103*) as *XfFah* and *XfAah*. The surplus in any given year is allocated within *Xf* columns as follows:

77.a $\mathbf{rSAM4}^{\tilde{\tau}}(FC,Xf\lambda ah) = [\{\mathbf{\Omega}_{1,FC}\times\mathbf{rSAM3}^{\tilde{\tau}}(FC,Xf\lambda ah)\times\mathbf{\Omega}_{Xf\lambda ah,1}\}^{-1}\times\lambda\mathrm{AH}]$

77.b $\times\ \mathbf{rSAM3}^{\tilde{\tau}}(FC,Xf\lambda ah),\ \lambda \in \{F,A\}$

78.a $\mathrm{shr}\lambda afh^{\tilde{\tau}} = \mathbf{\Omega}_{1,FC}\times[\mathbf{rSAM3}^{\tilde{\tau}}(FC,Xf\lambda ah)\mathbf{-rSAM4}^{\tilde{\tau}}(FC,Xf\lambda ah)] \times \mathbf{\Omega}_{Xf\lambda ah,1}$

78.b $\times\ \{\lambda\mathrm{AFH} - \mathbf{\Omega}_{1,FC}\times\mathbf{rSAM3}^{\tilde{\tau}}(FC,Xf\lambda afh) \times \mathbf{\Omega}_{Xf\lambda afh,1}\}^{-1},\ \lambda \in (F,A)$

79.a $\mathbf{rSAM4}^{\tilde{\tau}}(FC,Xf\lambda afh) = \mathbf{rSAM3}^{\tilde{\tau}}(FC,Xf\lambda afh) +$

79.b $(1 - \{\mathbf{\Omega}_{1,FC}\times\mathbf{rSAM3}^{\tilde{\tau}}(FC,Xf\lambda ah)\times\mathbf{\Omega}_{Xf\lambda ah,1}\}^{-1}\times\lambda\mathrm{AH}) \times$

79.c $(\mathbf{rSAM3}^{\tilde{\tau}}(FC,Xf\lambda ah)\times\mathbf{\Omega}_{Xf\lambda ah,1}) \times$

79.d $\min(1,\mathrm{shr}\lambda afh^{\tilde{\tau}}) \times$

79.e $(\mathbf{\Omega}_{1,FC}\times\mathbf{rSAM3}^{\tilde{\tau}}(FC,Xf\lambda afh))\times\{\mathbf{\Omega}_{1,FC}\times\mathbf{rSAM3}^{\tilde{\tau}}(FC,Xf\lambda afh)\times\mathbf{\Omega}_{Xf\lambda afh,1})\}^{-1},$

79.f $\lambda \in (F,A)$

80.a $\mathbf{rSAM4}^{\tilde{\tau}}(FC,Xh) = \mathbf{rSAM3}^{\tilde{\tau}}(FC,Xh) + (1 - \{\mathbf{\Omega}_{1,FC}\times\mathbf{rSAM3}^{\tilde{\tau}}(FC,Xf\lambda ah)\times\mathbf{\Omega}_{Xf\lambda ah,1}\}^{-1}\times\lambda\mathrm{AH})$

80.b $\times\ (\mathbf{rSAM3}^{\tilde{\tau}}(FC,Xf\lambda ah)\times\mathbf{\Omega}_{Xf\lambda ah,1}) \times [1- \min(1,\mathrm{shr}\lambda afh^{\tilde{\tau}})],\ \lambda \in (F,A)$

In equation 77, a scaling down of all relevant food and alcohol expenditure columns was based on the ratio of USDA, ERS to BEA totals reported in the first two data columns of table 6 (i.e., expression within squared brackets [ ] of equation 77). These surplus commodities were misallocated due to the passthrough accounting and should be reallocated to foodservice marketing channels. To facilitate this reallocation, we must measure the value of the USDA, ERS' FAFH and AAFH series not captured in the BEA PCE series represented by the *XfFafh* and *XfAafh* subaccounts and determine what share of this deficit is captured by the surplus measured in equation 77. This share statistic is compiled in equation 78, where equation line 78.a measures the total value of the FAH and AAH surplus and equation line 78.b measures the corresponding FAFH and AAFH deficit. This ratio is denoted shr$Fafh^{\tilde{\tau}}$ and shr$Aafh^{\tilde{\tau}}$, respectively. As shown in table 6, the FAH surplus is typically less than the FAFH deficit, implying a share statistic between 0 and 1. However, in some years (mostly in the 1990s), the surplus exceeded the deficit implying a share statistic greater than 1. For alcohol, the AAH surplus always exceeded the AAFH deficit. In 2020, there was an AAFH surplus.[48] The relevant subaccounts for these steps are the food and alcohol spending by marketing channel as identified in table 5 (*Xf1201–Xf1203* and *Xf2201–Xf2203*) and denoted as *XfFafh* and *XfAafh*.

Equation 79 addresses the reallocation of the FAH and AAH surplus. From our initial measures of *XfFafh* and *XfAafh* (right side of equation 79.a), we add the product of equations 79.b and 79.c, which consolidates the FAH and AAH surplus across all FAH and AAH tables to a single column for each. Equation 79.d scales this vector down by the share measure from equation 78 (i.e., changing all share measures > 1 to a value of 1) to ensure the reallocation does not exceed the FAFH and AAFH deficits.

[47] We excluded *Xf1126* (i.e., food consumed on farms) since passthrough accounting and other factors are not related to this measure.

[48] For this one year we address the surplus by adapting equation 77 for *AAFH* and update equation 80 accordingly.

Equation 79.e distributes this surplus to each FAFH and AAFH marketing channel in proportion to their market shares. In equation 80, all remaining FAH and AAH surplus (most common for AAH) is reallocated to the *XIh* column to ensure the overall account remains balanced.

Meals and Soft Drinks at Restaurants (FAFH) and Total Alcohol Away From Home (AAFH)

For FAFH in most years, there remained a substantial deficit and this discrepancy could be addressed using the food-at-work vouchers. Rather than allocate these vouchers to three *XfFafh* accounts, we redirected all vouchers for meals at work (*v01*; table 4) to the food and nonalcoholic beverages at work PCE subaccount (*Xf1207*; table 5) denoted *XfFaw*:

81.a $\mathbf{rSAM4}^{\tilde{\tau}}(FC,XfFaw) = [\text{FAFH} - \mathbf{\Omega}_{1,FC}\times\mathbf{rSAM4}^{\tilde{\tau}}(FC,XfFafh)\times\mathbf{\Omega}_{XfFafh,1}]$

81.b $\times\ [\mathbf{rSAM4}^{\tilde{\tau}}(FC,v01)\times\{\ \mathbf{\Omega}_{1,FC}\times\mathbf{rSAM4}^{\tilde{\tau}}(FC,v01)\times\mathbf{\Omega}_{v01,1}\}^{-1}]$

82. $\mathbf{rSAM4}^{\tilde{\tau}}(FC,v01) = \mathbf{rSAM4}^{\tilde{\tau}}(FC,v01) - \mathbf{rSAM4}^{\tilde{\tau}}(FC,XfFaw)\times\mathbf{\Omega}_{XfFaw,v01}$

83. $\mathbf{rSAM4}^{\tilde{\tau}}(Fk,v01) = \mathbf{rSAM4}^{\tilde{\tau}}(Fk,v01) + \mathbf{\Omega}_{Fk,FC}\times\mathbf{rSAM4}^{\tilde{\tau}}(FC,XfFaw)\times\mathbf{\Omega}_{XfFaw,v01}$

84. $\mathbf{rSAM4}^{\tilde{\tau}}(Fk,XfFaw) = \mathbf{rSAM4}^{\tilde{\tau}}(Fk,XfFaw) - \mathbf{\Omega}_{Fk,FC}\times\mathbf{rSAM4}^{\tilde{\tau}}(FC,XfFaw)$

The expression to the right on equation 81.a represents the dollar value of the remaining discrepancy between USDA, ERS and BEA FAFH measures. In equation 81.b the value in 81.a is distributed among caloric (*cal*) and foodservice (*fs*) commodity outlays in the same proportions as they are assembled in the *v01* account. Implicit in these calculations is that the total market value of this voucher group exceeds the dollar value of the remaining discrepancy. Because this condition is always true over the entire time series, we avoided including additional notation by omitting any conditional statement that would require an alternative calculation if the condition were violated. We also omitted similar equations and notation for alcoholic beverages because discrepancies in all years are fully addressed by application of equations 77–80. The commodity acquisitions redirected from the *v01* column account are deducted from that account in equation 82. This voucher column account now has a surplus of sales from the voucher row account. Equations 83 and 84 assigned this surplus to the voucher operating surplus row (*FLk*) and subsequently transferred this surplus to the operating surplus row of the *XfFaw* final demand column. This step ensured that total GDI and GDP remain unaltered, equal, and that the overall account remains balanced.

School Meals and Other Institutional Furnished Food and Beverages

For school meals in most years, there also remained a substantial deficit and this discrepancy was addressed using the school meal vouchers. Although BEA does measure PCE outlays on foodservices for both schools and colleges, far more dollars were recorded for outlays on food and beverages by governments and education service activities, which we redirected to the school meals voucher account. To address the deficit in the former measure, we redirected all or part of vouchers for school meals (*v04*; table 4) to the school meals and nonalcoholic beverages subaccount (*Xf1204*; table 5) denoted *XfSm*:

85.a $\mathbf{rSAM4}^{\tilde{\tau}}(FCV,XfSm) = \mathbf{rSAM4}^{\tilde{\tau}}(FCV,XfSm) + [\text{SM} - \mathbf{\Omega}_{1,FCV}\times\mathbf{rSAM3}^{\tilde{\tau}}(FCV,XfSm)\times\mathbf{\Omega}_{XfSm,1}]$

85. $\times\ [\mathbf{rSAM4}^{\tilde{\tau}}(FCV,v04)\times\{\ \mathbf{\Omega}_{1,FCV}\times\mathbf{rSAM4}^{\tilde{\tau}}(FCV,v04)\times\mathbf{\Omega}_{v04,1}\}^{-1}]$

86. $\mathbf{rSAM4}^{\tilde{\tau}}(FCV,v04) = \mathbf{rSAM4}^{\tilde{\tau}}(FCV,v04) - [\mathbf{rSAM4}^{\tilde{\tau}}(FCV,XfSm) - \mathbf{rSAM3}^{\tilde{\tau}}(FCV,XfSm)]\times\mathbf{\Omega}_{XfSm,v04}$

87.a $\mathbf{rSAM4}^{\tilde{\tau}}(Fk,v04) = \mathbf{rSAM4}^{\tilde{\tau}}(Fk,v04)$

87.b $+\ \mathbf{\Omega}_{Fk,FCV}\times[\mathbf{rSAM4}^{\tilde{\tau}}(FCV,XfSm) - \mathbf{rSAM3}^{\tilde{\tau}}(FCV,XfSm)]\times\mathbf{\Omega}_{XfSm,v04}$

88. $\mathbf{rSAM4}^{\tilde{\tau}}(Fk,XfSm) = \mathbf{rSAM4}^{\tilde{\tau}}(Fk,XfSm) - \mathbf{\Omega}_{Fk,FCV}\times[\mathbf{rSAM4}^{\tilde{\tau}}(FCV,XfSm) - \mathbf{rSAM3}^{\tilde{\tau}}(FCV,XfSm)]$

The expression to the right of the equality on equation 85.a adds the dollar value of the discrepancy between USDA, ERS' and BEA's school meal measures. In equation 85.b this discrepancy is distributed to outlays among caloric (*cal*) commodities and overhead cost vouchers (*vo1*) in the same proportions as they are assembled in the *v04* account. The total market value of this voucher group exceeds the dollar value of the remaining discrepancy has remained implicit in these calculations. Because this condition is always true over the entire time series, we avoided including additional notation by omitting any conditional statement that would require an alternative calculation if violated. We note that potential future deficits in this voucher availability could be addressed by expanding the voucher overhead costs to include operating surplus since the host activities for the school meal vouchers extend beyond labor costs. The commodity acquisitions redirected from the *v04* column account are deducted from that account in equation 86. This voucher column account now has a surplus of sales from the voucher row account. Equations 87 and 88 assigned this surplus to the voucher operating surplus row (*FLk*) and subsequently transfers this surplus to the operating surplus row of the *XfSm* final demand column. This step has kept the attribution of GDP to the final demand for the host activities and ensured that the overall account remains balanced while allowing for the identification of these important food dollars.

For meals at other institutions and furnished meals, we have partial measures in the BEA's PCE food furnished to employees category. The numerous activities whose outlays on food and beverages that are redirected to the institutional furnished meals and food-assistance (*v03*) voucher account have more dollars recorded. To address the deficit in the former measure, we redirected all or part of the *v03* vouchers to the institutional and employer furnished meals plus food assistance subaccount (*Xf1206*; table 5) denoted *XfIefa*:

89.a $$\mathbf{rSAM4}^{\tilde{\tau}}(FCV,XfIefa) = \mathbf{rSAM4}^{\tilde{\tau}}(FCV,XfIefa) + [OIFM - \mathbf{\Omega}_{1,FCV}\times\mathbf{rSAM3}^{\tilde{\tau}}(FCV,XfIefa)\times\mathbf{\Omega}_{XfIefa,1}]$$

89.b $$\times [\mathbf{rSAM4}^{\tilde{\tau}}(FCV,v03) \times \{ \mathbf{\Omega}_{1,FCV}\times\mathbf{rSAM4}^{\tilde{\tau}}(FCV,v03)\times\mathbf{\Omega}_{v03,1}\}^{-1}]$$

90.a $$\mathbf{rSAM4}^{\tilde{\tau}}(FCV,v03) = \mathbf{rSAM4}^{\tilde{\tau}}(FCV,v03)$$

90.b $$- [\mathbf{rSAM4}^{\tilde{\tau}}(FCV,XfIefa) - \mathbf{rSAM3}^{\tilde{\tau}}(FCV,XfIefa)] \times \mathbf{\Omega}_{XfIefa,v03}$$

91.a $$\mathbf{rSAM4}^{\tilde{\tau}}(Fk,v03) = \mathbf{rSAM4}^{\tilde{\tau}}(Fk,v03)$$

91.b $$+ \mathbf{\Omega}_{Fk,FCV} \times [\mathbf{rSAM4}^{\tilde{\tau}}(FCV,XfIefa) - \mathbf{rSAM3}^{\tilde{\tau}}(FCV,XfIefa)] \times \mathbf{\Omega}_{XfIefa,v03}$$

92. $$\mathbf{rSAM4}^{\tilde{\tau}}(Fk,XfIefa) = \mathbf{rSAM4}^{\tilde{\tau}}(Fk,XfIefa) - \mathbf{\Omega}_{Fk,FCV} \times [\mathbf{rSAM4}^{\tilde{\tau}}(FCV,XfIefa) - \mathbf{rSAM3}^{\tilde{\tau}}(FCV,XfIefa)]$$

The expression to the right in equation 89.a adds the dollar value of the discrepancy between USDA, ERS' and BEA's other institutions and furnished meals measures. In equation 89.b, this discrepancy is distributed to outlays among caloric (*cal*) commodities and overhead cost vouchers (*vo1*) in the same proportions as they are assembled in the *v03* account. The total market value of this voucher group exceeds the dollar value of the remaining discrepancy. Because this condition is always true over the entire time series, we avoided including additional notation by omitting any conditional statement, which would require an alternative calculation if violated. We note that potential future deficits in this voucher availability could be addressed by including the employer furnished food and beverage voucher (*v02*). The commodity acquisitions redirected from the *v03* column account are deducted from that account in equation 90. This voucher column account now has a surplus of sales from the voucher row account. Equations 91 and 92 assigned this surplus to the voucher operating surplus row (*FLk*) and subsequently transfers this surplus to the operating surplus row of the *XfIefa* final demand column. This step keeps the attribution of GDP to the final demand for the host activities and ensures that the overall account remains balanced while allowing for the identification of these important food dollars.

Incorporating USDA, Economic Research Service Price Spreads Data Products

Equations 93–116 incorporate USDA, ERS farm-to-retail price spread data and corresponding commodity procurement voucher accounts for retail beef (*psb; v06*), pork (*psp; v07*), whole milk (*psm; v09*), fresh fruits (*psf; v11*), and fresh vegetables (*psv; v12*), plus residual procurements for other meat animals for slaughter (*psom; v08*), and raw milk for nonmilk retail uses of fluid milk, and other dairy (*psod; v10*):

- All meat animal procurements by the animal slaughter (except poultry) activity (*Amp*) are moved to the beef (*v06*), pork (*v07*), and other meat animal (*v09*) procurement voucher accounts in equations 91–92, and production for the primary commodity (*mp*) of the *Amp* activity is reduced by the producer value of the meat animals no longer being procured in equation 93.
- What remains is a slaughtering service that must be combined with procured meat animal vouchers for retail sales, and this split among food-at-home sales of beef (*Xf1103*), pork (*Xf1104*), and other meats (*Xf1105*) is compiled in equations 94–95.
- Remaining buyers of meat processing services (*mp*) in **rSAM4** will claim a portion of remaining procured meat animals in proportion to their purchase of meat processing services, as outlined in equation 96, and their recorded outlays on *mp* are reduced by the value of procured animal purchases as outlined in equation 97.
- All dairy milk procurement by the fluid milk and butter processing activity (*a045*) is moved to the retail milk (*v09*), and other dairy products (*v10*) procurement voucher accounts in equations 98–99, and production for the primary commodity of the fluid milk and butter activity (*c045*) is reduced by the producer value the raw milk no longer being procured in equation 100.
- What remains is a raw milk processing service that must be combined with procured raw milk vouchers for retail sales, and this split among food-at-home sales of fluid milk and other dairy products, plus intermediate sales to activities purchasing *c045* is compiled in equations 101–104.
- For retail sales of fresh fruits and vegetables, our survey of research and data indicate that passthrough accounting has a more pronounced under allocation to foodservice among these markets, and perhaps for the same reason understates the transportation costs of marketing fresh produce. USDA, ERS price spread data correct under allocations to foodservices, and return freight costs from foodservices (same equations are applied to fruit [*105f–114f*] and vegetables [*105v–114v*]):
  - Equation 105 applies fruit and vegetable price spreads to retail values to measure fresh fruit and vegetable procurement and assigns these to voucher accounts *v11* and *v12*.
  - Equations 106–107 measure surplus produce for directing to foodservices, and measure market shares among the three foodservice marketing channels for claims of produce.
  - Equation 108 uses parameters in equations 106–107 to allocate produce to foodservices.
  - In equation 113, transportation costs reclaimed from foodservices of equal value as produce sent to those services (equation 112), plus wholesale and retail costs reported in **PCEb** and located in column accounts *Xf1113* and *Xf1114* (equation 110) are all moved to voucher accounts *v11* and *v12* that already have produced procurements (equation 105).
  - In equation 114, the fruit and vegetable at home accounts (*Xf1113* and *Xf1114*) that were zeroed-out sequentially in equations 109 and 111 are replenished with outlays on the entire produce voucher accounts (*v11* and *v12*), which preserves total food-at-home outlays reported in **PCEb** but redistributes producer values between fresh produce and transportation services.

- This subsection largely involves movements of outlays between farm commodity procurement voucher accounts and the *Xf* accounts depicted in the columns and rows of figure 5.

The USDA, ERS Price Spreads from Farm to Consumer (USDA, ERS, 2024b) and Meat Price Spreads (USDA, ERS, 2024c) data products (denoted as PSF2C) measure the farmers' share of retail costs for several farm commodities. Conceptually, they measure the farm proceeds that result from farm food commodity sales on a per retail dollar basis, accounting for any food loss between farm sales and points of purchase. An important difference between these measures and the farm share measure of USDA, ERS' Food Dollar Marketing Bill is that the latter measures all farm sales, or the net of farm-to-farm transactions, linked to an average retail dollar spent on food commodities. For example, for each retail dollar spent on fresh apples at the grocery store, both the marketing bill series in the Food Dollar data product and the PSF2C series measures the farm value of all apple sales entering the retail marketing channel, not just the apples ultimately purchased by retail consumers.[49] However, the Food Dollars' marketing bill also measures any farm sales of feedstock used for the production of biofuels that help power the transportation of these apples from farms to point of retail purchase, as well as all other farm sales supporting post farmgate processes that facilitate the retail apple sales.

Based on these definitions and the commodity coverage of the PSF2C data products, we can incorporate price spread measures for the following commodities: (1) beef (*psb*), (2) pork (*psp*), (3) whole milk (*psm*), (4) fresh fruits (*psf*), and (5) fresh vegetables (*psv*). Using PSF2C data for the three animal products allowed us to conduct an explicit accounting of residual procurements for other meat animals for slaughter (*psom*) and raw milk for nonmilk retail uses of fluid milk or other dairy (*psod*). The PSF2C data are used differently for each commodity.

Beef, Pork, and Other Meats, Excluding Poultry

In the national accounts, all meat animals marketed through the food retail and foodservice marketing channels pass through the animal slaughter activity. Poultry passes through the poultry processing activity and all other meats pass through an animal slaughter, rendering, and processing activity, or meat processing for short. Fish and seafood not marketed as fresh passes through a seafood product preparation and packaging activity, or seafood processing for short. In the BEA's detail accounts and our *F* level accounts based on these accounts, all beef, pork, and other meat retail purchases are recorded as direct purchases of the meat processing commodity, except for a small amount of other meat (i.e., game meat) purchases of the fishing, hunting, and trapping commodity. A multiplier model using this data configuration would lead to the same mix of meat animal slaughter when purchasing beef, pork, or other meats. For example, the farm value of animals slaughtered per retail dollar of beef, pork, or other meats comprised of 70 percent beef cattle, 29 percent animals except beef and poultry, and 1 percent hunting and trapping based on the 2017 detailed BEA use table (USDOC, BEA, 2024c).

We used the PSF2C data and additional voucher accounts to align animal procurements to retail markets by type of meats purchased. First, we decoupled the procurement of animals for slaughter from the meat processing activity/commodity (*Amp/mp*), where animal commodities entering this supply chain are beef cattle (*c008*) and animal production except cattle, poultry, and eggs (*c010*). We introduced three additional voucher accounts (i.e., beef cattle procurement for slaughter (*v06*), pork animal procurement for retail supply (*v07*), and other animal procurement for slaughter (*v08*)) as follows:

[49] For example, if a grower sold one bushel of apples directly to a local retailer for $30.00 and the retailer was only able to sell a half bushel for $60.00 due to damage that made the other half unsellable, the farm share on a per dollar basis would be $0.50 (30.00/60.00).

93b. $\mathbf{rSAM4}^{\tilde{\tau}}(c008,v06) = \mathbf{rSAM4}^{\tilde{\tau}}(c008,Amp)$

94b. $\mathbf{rSAM4}^{\tilde{\tau}}(c008,Amp) = 0$

93p. $\mathbf{rSAM4}^{\tilde{\tau}}(c010,v07) = \mathrm{psp}^{\tilde{\tau}} \times \mathbf{\Omega}_{c010,FCV} \times \mathbf{rSAM4}^{\tilde{\tau}}(FCV,Xf1104) \times \mathbf{\Omega}_{Xf1104,v07}$

94p. $\mathbf{rSAM4}^{\tilde{\tau}}(c010,Amp) = \mathbf{rSAM4}^{\tilde{\tau}}(c010,Amp) - \mathbf{rSAM4}^{\tilde{\tau}}(c010,v07) \times \mathbf{\Omega}_{v07,Amp}$

93om. $\mathbf{rSAM4}^{\tilde{\tau}}(c010,v08) = \mathbf{rSAM4}^{\tilde{\tau}}(c010,Amp) \times \mathbf{\Omega}_{Amp,v08}$

94om. $\mathbf{rSAM4}^{\tilde{\tau}}(c010,Amp) = 0$

95.a $\mathbf{rSAM4}^{\tilde{\tau}}(Amp,mp) = \mathbf{rSAM4}^{\tilde{\tau}}(Amp,mp) - \mathbf{\Omega}_{Amp,c008} \times \mathbf{rSAM4}^{\tilde{\tau}}(c008,v06) \times \mathbf{\Omega}_{v06,mp}$

95.b $- \mathbf{\Omega}_{Amp,c010} \times (\mathbf{rSAM4}^{\tilde{\tau}}(c010,v07) \times \mathbf{\Omega}_{v07,mp} + \mathbf{rSAM4}^{\tilde{\tau}}(c010,v08) \times \mathbf{\Omega}_{v08,mp})$

It is important that the above equations are executed in sequence. In equation 93b, all procurement of beef cattle is reassigned to the *v06* voucher account and, thus, is zeroed out of the meat processing activity in equation 94b. For pork we first determined the farm value of hogs and pigs in total annual retail pork sales. This is measured in equation 93p as the product of the pork farm share ($\mathrm{psp}^{\tilde{\tau}}$) and the total retail value of food-at-home pork sales and is recorded as a purchase of animal production except cattle and poultry (*c010*). These identified pork animal sales are deducted from the *c010* outlays of the meat processing industry, as stated in equation 94p. After this deduction, equation 93om reassigns all remaining *c010* animal procurement to the *v08* voucher account and is also zeroed out of the meat processing activity in equation 94om. Now that all meat animal procurement has been entirely outsourced from the meat processing activities to the three meat animal procurement accounts, equation 95 scales back assembly of the meat processing commodity from output of the meat processing activity by a value equal to the total value of meat animal procurement from the three animal procurement voucher accounts (*v06*–*v08*).

All outlays on the meat processing commodity in our accounts now must be split between meat processing and meat animal procurements. Our approach for the PCE outlays on beef, pork, and other meats (*Xf1103* to *Xf1105*) aligned the type of meat animal with the type of expenditure:

96b. $\mathbf{rSAM4}^{\tilde{\tau}}(v06,Xf1103) = \mathrm{psb}^{\tilde{\tau}} \times \mathbf{\Omega}_{v06,FCV} \times \mathbf{rSAM4}^{\tilde{\tau}}(FCV,Xf1103)$

97b. $\mathbf{rSAM4}^{\tilde{\tau}}(mp,Xf1103) = \mathbf{rSAM4}^{\tilde{\tau}}(mp,Xf1103) - \mathbf{\Omega}_{mp,v06} \times \mathbf{rSAM4}^{\tilde{\tau}}(v06,Xf1103)$

96p. $\mathbf{rSAM4}^{\tilde{\tau}}(v07,Xf1104) = \mathrm{psp}^{\tilde{\tau}} \times \mathbf{\Omega}_{v07,FCV} \times \mathbf{rSAM4}^{\tilde{\tau}}(FCV,Xf1104)$

97p. $\mathbf{rSAM4}^{\tilde{\tau}}(mp,Xf1104) = \mathbf{rSAM4}^{\tilde{\tau}}(mp,Xf1104) - \mathbf{\Omega}_{mp,v07} \times \mathbf{rSAM4}^{\tilde{\tau}}(v07,Xf1104)$

96om. $\mathbf{rSAM4}^{\tilde{\tau}}(v08,Xf1105) = \mathrm{psp}^{\tilde{\tau}} \times \mathbf{\Omega}_{v08,FCV} \times \mathbf{rSAM4}^{\tilde{\tau}}(FCV,Xf1105)$

97om. $\mathbf{rSAM4}^{\tilde{\tau}}(mp,Xf1105) = \mathbf{rSAM4}^{\tilde{\tau}}(mp,Xf1105) - \mathbf{\Omega}_{mp,v08} \times \mathbf{rSAM4}^{\tilde{\tau}}(v08,Xf1105)$

For beef and pork, retail outlays for meat animal procurements are measured in equations 96b and 96p, respectively, where their corresponding total retail dollar measures are scaled by their respective farm-to-retail price spread measures, $\mathrm{psb}^{\tilde{\tau}}$ and $\mathrm{psp}^{\tilde{\tau}}$, respectively. For the other meats retail market, we did not have farm to retail price spread information. Our approach was to apply the pork price spread parameter ($\mathrm{psp}^{\tilde{\tau}}$) to this retail market as measured in equation 96om.[50] For beef retail outlays, the vouchers procure beef cattle, while for pork and other meat retail outlays the vouchers procure the other meat animals. For all three markets, the value of these animal procurements was deducted from the outlays on meat

[50] The other available alternative is to assume equal farm values for all remaining transactions as we do below after assigning our other meats farm value. This would imply a farm share measure between the beef and pork measures. Our assessment is that the pork measure is more representative of the retail market for other meats.

processing in equations 97b, 97p, and 97om for beef, pork, and other meats respectively, due to knowledge that all meat animal procurement by meat processors was outsourced in equations 93 and 94.

Outlays for remaining meat animal procurements not allocated above were assumed to be proportional to the available remaining meat animals and get distributed in proportion to outlays on meat processing by all activities and institutions not already addressed above. We also excluded outlays on meat processing services by the meat processing industry since it has no claim on the unallocated meat animals due to its outsourcing of such procurement, so the set of remaining buyers of meat processing services with a claim on meat animals is denoted *AxmpXxmeat* (activities except meat processing, and nonmeat PCE final commodity demand):

98λ.a $\mathbf{rSAM4}^{\tilde{\tau}}(\lambda, AxmpXxmeat) = (\ \mathbf{\Omega}_{\lambda,FCV} \times \mathbf{rSAM4}^{\tilde{\tau}}(FCV,\lambda) \times \mathbf{\Omega}_{\lambda,AxmpXxmeat}$

98λ.b $-\ \mathbf{rSAM4}^{\tilde{\tau}}(\lambda, Xmeat) \times \mathbf{\Omega}_{Xmeat,AxmpXmeat})$

98λ.c $\times\ (\ \mathbf{\Omega}_{1,mp} \times \mathbf{rSAM4}^{\tilde{\tau}}(mp, AxmpXmeat)$

98λ.d $\times\ \{\mathbf{\Omega}_{1,mp} \times [\mathbf{rSAM4}^{\tilde{\tau}}(mp, AxmpXmeat) \times \mathbf{\Omega}_{AxmpXmeat,1}]\}^{-1}\ )''\ ,\ \lambda \in (v06, v08)$

99λ.a $\mathbf{rSAM4}^{\tilde{\tau}}(mp, AxmpXxmeat) = \mathbf{rSAM4}^{\tilde{\tau}}(mp, AxmpXxmeat) - \mathbf{\Omega}_{\mathrm{mp},\lambda} \times \mathbf{rSAM4}^{\tilde{\tau}}(\lambda, AxmpXxmeat),$

99λ.b $\lambda \in (v06, v08)$

For each voucher market, λ ∈ (*v06,v08*), we can subtract meat animal procurement voucher allocations to meat PCE markets (as measured in equation 98λ.b) from the pool of total vouchers of type λ (as measured in equation line 98λ.a) to measure the total remaining unallocated type λ vouchers. Equations 98λ.c and 98λ.d allocate the remaining unallocated vouchers to all activities and final markets with meat processing outlays that are identified in set *AxmpXxmeat* in proportion to each market's share of their total outlays on meat processing (*mp*). In other words, we assumed the value of meat processing services per farm value of meat animal is the same for all remaining animals subject to meat processing. Like all other allocations, each activity and final meat animal voucher deduct their value from the outlays on meat processing in equation 99 because all meat animal procurement by meat processors was outsourced in equations 93 and 94.

Fresh Milk and Dairy Products

In the national accounts, all fresh milk consumption expenditures purchased through retail marketing channels involve the purchases of fluid milk and butter (*c045*) produced mainly by the fluid milk and butter manufacturing activity (*a045*). This commodity is also marketed in other retail marketing channels (e.g., processed dairy products and fats and oils) and is also purchased as an intermediate product for activities such as cheese manufacturing and for school lunches. A multiplier model using this data configuration would distribute the farm value of milk production used in fluid milk and butter manufacturing to all uses of the latter in proportion to each purchaser share of expenditures on fluid milk and butter manufacturing. This is a strong assumption. For example, when a fluid milk and butter manufacturer take purchase orders for the same dollar amount from one retailer selling this product as whole milk, a second retailer selling this product as butter, and a cheese manufacture using this product in the production of cheese, it does not seem likely that the product sold to each buyer has the same farm value for the fresh milk embodied in each purchase.

While we did not have farm value measures for all such fluid milk and butter manufacturers customers, we did know the annual retail value of fluid milk expenditures from BEA and the annual average farm value of milk per dollar of retail whole milk expenditures from USDA, ERS. We used this knowledge to assign the farm value for retail milk sales before reverting to the multiplier model approach to distribute values to all other uses. Whole milk farm values are likely to be more representative of average farm values for all retail sales of fresh milk than using a single approach for all users. We used the whole milk price spread parameter ($\text{psm}^{\tilde{\tau}}$) for procurement of raw milk for retail fluid milk supply (*v09*), and the remaining procurement of raw milk by fluid milk and butter manufacturers is for sales to other dairy product buyers (*v10*):

100m. $\mathbf{rSAM4}^{\tilde{\tau}}(c007,v09) = \text{psm}^{\tilde{\tau}} \times \mathbf{\Omega}_{c007,FCV} \times \mathbf{rSAM4}^{\tilde{\tau}}(FCV,Xf1108) \times \mathbf{\Omega}_{Xf1108,v09}$

101m. $\mathbf{rSAM4}^{\tilde{\tau}}(c007,a045) = \mathbf{rSAM4}^{\tilde{\tau}}(c007,a045) - \mathbf{rSAM4}^{\tilde{\tau}}(c007,v09) \times \mathbf{\Omega}_{v09,a045}$

100od. $\mathbf{rSAM4}^{\tilde{\tau}}(c007,v10) = \mathbf{rSAM4}^{\tilde{\tau}}(c007,a045) \times \mathbf{\Omega}_{a045,v10}$

101od. $\mathbf{rSAM4}^{\tilde{\tau}}(c007,a045) = 0$

102.a $\mathbf{rSAM4}^{\tilde{\tau}}(a045,c045) = \mathbf{rSAM4}^{\tilde{\tau}}(a045,c045) - \mathbf{\Omega}_{a045,c007}$

102.b $\times (\mathbf{rSAM4}^{\tilde{\tau}}(c007,v09) \times \mathbf{\Omega}_{v09,c045} + \mathbf{rSAM4}^{\tilde{\tau}}(c007,v10) \times \mathbf{\Omega}_{v10,c045})$

It is important that the above equations are also executed in sequence. In equation 100m, we must first determine the farm value of milk production in total annual retail fresh milk sales. This is measured as the product of the whole milk price spread ($\text{psm}^{\tilde{\tau}}$) and the total retail value of food-at-home fresh milk sales and is recorded as a purchase of farm milk production (*c007*). These identified farm milk sales are deducted from the *c007* outlays of the fluid milk and butter processing activity, as stated in equation 100m. After this deduction, equation 100od reassigns all remaining *c007* farm milk procurement to the *v10* voucher account and thus is also zeroed out of the fluid milk and butter processing activity in equation 100od. Now that all farm milk procurements have been entirely outsourced from the fluid milk and butter processing activities to the two dairy procurement accounts, equation 102 scales back the fluid milk and butter processing commodity assembly from the output of the same activity by a value equal to the total value of farm milk procurement from the two dairy procurement voucher accounts (*v09–v10*).

All outlays on the fluid milk and butter processing commodity in our accounts now must be split between processing and the two dairy procurement vouchers. Our approach for the personal consumption expenditure (PCE) outlays on fresh milk (*Xf1103*) is different than for other markets:

103m. $\mathbf{rSAM4}^{\tilde{\tau}}(v09,Xf1108) = \text{psm}^{\tilde{\tau}} \times \mathbf{\Omega}_{v09,FCV} \times \mathbf{rSAM4}^{\tilde{\tau}}(FCV,Xf1108)$

104m. $\mathbf{rSAM4}^{\tilde{\tau}}(c045,Xf1108) = \mathbf{rSAM4}^{\tilde{\tau}}(c045,Xf1108) - \mathbf{\Omega}_{c045,v09} \times \mathbf{rSAM4}^{\tilde{\tau}}(v09,Xf1108)$

Retail outlays for the farm milk vouchers in the retail fresh milk market are measured in equation 103m, where their corresponding total retail dollars measure is scaled by the farm-to-retail price spread measure, $\text{psm}^{\tilde{\tau}}$. The value of this farm milk voucher procurement is deducted from the outlays on fluid milk processing in equation 104m, due to knowledge that all farm milk procurement by fluid milk processors was outsourced in equation 101m.

Outlays on farm milk procurements for other dairy markets (*v10*) are distributed in proportion to outlays on fluid milk and butter processing by all activities and institutions other than retail fresh milk PCE. We also excluded outlays on fluid milk and butter processing services by the fluid milk and butter processing activity since it has no claim due to its procurement outsourcing. The set of remaining buyers of milk and butter processing services with a claim on *v10* vouchers is denoted *Ax045Xxmilk* (activities except dairy processing and final commodity demands excluding milk PCE):

105.a $\mathbf{rSAM4}^{\tilde{\tau}}(v10,Ax045Xxmilk) = \mathbf{\Omega}_{v10,FCV} \times \mathbf{rSAM4}^{\tilde{\tau}}(FCV,v10) \times \mathbf{\Omega}_{v10,Ax045Xxmilk}$

105.b $\times (\mathbf{\Omega}_{1,c045} \times \mathbf{rSAM4}^{\tilde{\tau}}(c045,Ax045Xmilk)$

105.c $\times \{\mathbf{\Omega}_{1,c045} \times [\mathbf{rSAM4}^{\tilde{\tau}}(c045,Ax045Xmilk) \times \mathbf{\Omega}_{Ax045Xmilk,1}]\}^{-1})''$

106. $\mathbf{rSAM4}^{\tilde{\tau}}(\mathit{c045,Ax045Xxmilk}) = \mathbf{rSAM4}^{\tilde{\tau}}(\mathit{c045,Ax045Xxmilk}) - \mathbf{\Omega}_{\mathit{c045,v10}} \times \mathbf{rSAM4}^{\tilde{\tau}}(\mathit{v10,Ax045Xxmilk})$

Equation line 105.a allocates all other dairy procurement vouchers to all candidate markets. Equation lines 105.b and 105.c scales this allocation to all activities and final markets with milk and butter processing outlays identified in set *Ax045Xxmilk* in proportion to each's share of their total outlays on milk and butter processing (*c045*). That is, we assumed the value of milk and butter processing services per farm value of milk is the same for all remaining milk subjected to milk and butter processing. Like all other allocations, each activity and final other dairy vouchers deduct their value from the outlays on milk and butter processing in equation 106.

Fresh Fruits and Vegetables for Retail Sales

Incorporation of the meat and dairy price spread data products made no alterations to our PCE accounts. Rather, these data facilitated a redistribution of farm commodity flows along the various agri-food value chains in a zero-sum fashion such that any redirection of farm commodity flows among identified agri-food value chains was exactly offset by adjustments to either meat or diary processing services. In the case of the retail markets for fresh fruits and fresh vegetables, we must either ignore the USDA, ERS price spread data or expand the agri-food value chains we involved to incorporate them. This is because the fresh fruit and fresh vegetable retail transactions in our annual accounts have an exact correlation with the USDA, ERS price spread data for these agri-food value chain markets and they are consistently different. Although the $\mathbf{PCEb}^{\tau}$ tables from BEA consistently measured the farm share of the fresh fruit and fresh vegetable retail markets to range between 47–56 percent, empirical studies have shown substantially lower shares (Sexton et al., 2002; McLaughlin et al., 2015; USDA, ERS, 2023b; Census Bureau, 2024). These same empirical studies show that both wholesale and retail gross margins along fresh produce agri-food value chains are accurately reflected in BEA's $\mathbf{PCEb}^{\tau}$ tables from 1997–2017. McLaughlin et al. (2015) also indicated that transportation costs along fresh produce agri-food value chains paid by wholesalers and retailers were roughly on par with this produce's farm value. BEA's $\mathbf{PCEb}^{\tau}$ tables from 2007–17 have indicated a substantially small cost, ranging from 4.5–5.5 percent of the retail price.

Our research on these markets informed our formulation of how we can reconcile these discrepancies and incorporate the price spread data into our accounts. We hypothesized that the passthrough accounting that misdirected caloric commodities intended for foodservices to the retail marketing channel was more pronounced along fresh produce's agri-food value chains. This would imply that our approach of redirecting caloric commodities in proportion to their market values under allocates the movement of fresh produce to the foodservice marketing channels. We used USDA, ERS' price spread data to correct these under allocations and return transportation costs to these retail markets to preserve the expenditure levels of all marketing channels (i.e., denote *veg* the set of farm sourced vegetable commodities and note *c004* is the sole farm sourced fruit commodity):

107f. $\mathbf{rSAM4}^{\tilde{\tau}}(\mathit{c004,v11}) = \mathrm{psf}^{\tilde{\tau}} \times \mathbf{\Omega}_{\mathit{c004,FCV}} \times \mathbf{rSAM4}^{\tilde{\tau}}(\mathit{FCV,Xf1113}) \times \mathbf{\Omega}_{\mathit{Xf1113,v11}}$

108f. $\mathrm{XtraFrt}^{\tilde{\tau}} = \mathbf{\Omega}_{\mathit{1,FCV}} \times \mathbf{rSAM4}^{\tilde{\tau}}(\mathit{FCV,Xf1113}) - \mathbf{rSAM4}^{\tilde{\tau}}(\mathit{c004,v11})$

109f. $\mathbf{fsf}^{\tilde{\tau}}(\mathit{XfFafh}) = \mathbf{\Omega}_{\mathit{XfFafh,Afsc}} \times (\mathbf{rSAM4}^{\tilde{\tau}}(\mathit{c004,Afsc}) \times \{\mathbf{rSAM4}^{\tilde{\tau}}(\mathit{c004,Afsc}) \times \mathbf{\Omega}_{\mathit{Afsc,1}}\}^{-1})'$

110f. $\mathbf{rSAM4}^{\tilde{\tau}}(\mathit{c004,XfFafh}) = \mathbf{rSAM4}^{\tilde{\tau}}(\mathit{c004,XfFafh}) + \mathrm{XtraFrt}^{\tilde{\tau}} \times \mathbf{fsf}^{\tilde{\tau}}(\mathit{XfFafh})'$

111f. $\mathbf{rSAM4}^{\tilde{\tau}}(\mathit{c004,Xf1113}) = 0$

112f. $\mathbf{rSAM4}^{\tilde{\tau}}(\mathit{FC,v11}) = \mathbf{rSAM4}^{\tilde{\tau}}(\mathit{FC,v11}) + \mathbf{rSAM4}^{\tilde{\tau}}(\mathit{FC,Xf1113})$

113f. $\mathbf{rSAM4}^{\tilde{\tau}}(\mathit{FC,Xf1113}) = 0$

114f.a $\mathbf{rSAM4}^{\tilde{\tau}}(Ftr,v11) = \mathbf{rSAM4}^{\tilde{\tau}}(Ftr,v11) + \text{XtraFrt}^{\tilde{\tau}}$

114f.b $\times\ [\mathbf{rSAM4}^{\tilde{\tau}}(Ftr,XfFafh) \times \mathbf{\Omega}_{XfFafh,1} \times \{\mathbf{\Omega}_{1,Ftr} \times \mathbf{rSAM4}^{\tilde{\tau}}(Ftr,XfFafh) \times \mathbf{\Omega}_{XfFafh,1}\}^{-1}]$

115f.a $\mathbf{rSAM4}^{\tilde{\tau}}(Ftr,XfFafh) = \mathbf{rSAM4}^{\tilde{\tau}}(Ftr,XfFafh)$

115f.b $-\ \mathbf{rSAM4}^{\tilde{\tau}}(Ftr,XfFafh) \times \{[\mathbf{\Omega}_{1,Ftr} \times \mathbf{rSAM4}^{\tilde{\tau}}(Ftr,XfFafh)]''\}^{-1}$

115f.c $\times\ [\text{XtraFrt}^{\tilde{\tau}} \times \mathbf{fsf}^{\tilde{\tau}}(XfFafh)]''$

116f. $\mathbf{rSAM4}^{\tilde{\tau}}(v11,Xf1113) = \mathbf{\Omega}_{v11,FC} \times \mathbf{rSAM4}^{\tilde{\tau}}(FC,v11) \times \mathbf{\Omega}_{v11,Xf1113}$

These 10 equations incrementally incorporate USDA, ERS' annual price spread data for retail fresh fruit markets. Equation 107f measures the farm value of fresh fruit marketed to retail consumers as the product of USDA, ERS' annual farm share statistic ($\text{psf}^{\tilde{\tau}}$) and BEA's annual market value of fresh fruit sold in retail markets. This value was assigned as an outlay from voucher account *v11* on a segment of the fruit and tree nut commodities (*c004*) that represent fruit production for the domestic fresh fruit retail market. We introduced two new parameters in equations 108f and 109f that facilitate other steps in this reallocation process. The parameter $\text{XtraFrt}^{\tilde{\tau}}$ is compiled in equation 108f and is a scalar that measures the extra allocation of fresh fruit retail sales beyond those measured in equation 107f. The parameter $\mathbf{fsf}^{\tilde{\tau}}(XfFafh)$ in equation 109f is a three element vector measuring foodservice activity shares of total outlays on fresh fruit among the three foodservice activities (*Afsc*), which get mapped to the three food away from home marketing channels (*XfFafh*). The product of these two parameters distributes the surplus fresh fruits among the three foodservice marketing channels in equation 110f. We then zeroed out the original allocation of fresh fruit sales via food retail in equation 111f. Once these are zeroed out, equation 112f moves all remaining outlays on margin costs from the PCE subaccount (*Xf1113*) over to the fresh fruit for retail sales voucher subaccount (*v11*). This ensures the reallocation and assembly of outlays for the updated retail fresh fruit market is relegated to the voucher subaccounts where all such activities are consolidated. Once reallocated this value is zeroed out from its source in equation 113f. The value of the retail fresh fruit market is assumed to be unchanged, so an equal value of transportation services must be moved from the food-at-home marketing channel to the three food-away-from-home marketing channels (*XfFafh*) in amounts that equal the value of fresh fruits each received in a marketing channel. Additionally, the distribution of transportation services among transportation modes is replicated within the budget shares of each market. These transfers to the retail fresh fruits voucher account are achieved in equation 114f and deductions from their sources are specified in equation 115f. Lastly, the full value of the assembled retail fresh fruit vouchers is added to the zeroed-out PCE subaccount *Xf1113*. These zero-sum transfers facilitated by the voucher subaccount have preserved economywide balance of the **rSAM** accounts as defined in equations 9 and 10.

The same steps are repeated for the reallocation of fresh vegetables among retail and foodservice marketing channels, but there are three commodities that are sourced for fresh vegetables and this distinction requires additional parameters to achieve the desired reallocation:

107v.a $\mathbf{rSAM4}^{\tilde{\tau}}(veg,v12) = \text{psv}^{\tilde{\tau}} \times \mathbf{vshr}^{\tilde{\tau}}(veg) \times [\mathbf{\Omega}_{1,FCV} \times \mathbf{rSAM4}^{\tilde{\tau}}(FCV,Xf1114) \times \mathbf{\Omega}_{Xf1114,v12}]$,

107v.b where $\mathbf{vshr}^{\tilde{\tau}}(veg) = \mathbf{rSAM4}^{\tilde{\tau}}(veg,Xf1114) \times \{\mathbf{\Omega}_{1,FCV} \times \mathbf{rSAM4}^{\tilde{\tau}}(veg,Xf1114)\}^{-1}$

108v. $\text{XtraVeg}^{\tilde{\tau}} = \mathbf{\Omega}_{1,FCV} \times \mathbf{rSAM4}^{\tilde{\tau}}(FCV,Xf1114) - \mathbf{\Omega}_{1,veg} \times \mathbf{rSAM4}^{\tilde{\tau}}(veg,v12) \times \mathbf{\Omega}_{v12,Xf1114}$

109v. $\mathbf{FSV}^{\tilde{\tau}}(veg,XfFafh) = (\mathbf{rSAM4}^{\tilde{\tau}}(veg,Afsc) \times \{\mathbf{\Omega}_{1,veg} \times \mathbf{rSAM4}^{\tilde{\tau}}(veg,Afsc) \times \mathbf{\Omega}_{Afsc,1}\}^{-1}) \times \mathbf{\Omega}_{Afsc,XfFafh}$

110v. $\mathbf{rSAM4}^{\tilde{\tau}}(veg,XfFafh) = \mathbf{rSAM4}^{\tilde{\tau}}(veg,XfFafh) + \text{XtraVeg}^{\tilde{\tau}} \times \mathbf{FSV}^{\tilde{\tau}}(veg,XfFafh)$

111v. $\mathbf{rSAM4}^{\tilde{\tau}}(veg,Xf1114) = 0$

112v. $\mathbf{rSAM4}^{\tilde{\tau}}(FC,v12) = \mathbf{rSAM4}^{\tilde{\tau}}(FC,v12) + \mathbf{rSAM4}^{\tilde{\tau}}(FC,Xf1114) \times \boldsymbol{\Omega}_{Xf1114,v12}$

113v. $\mathbf{rSAM4}^{\tilde{\tau}}(FC,Xf1114) = 0$

114v.a $\mathbf{rSAM4}^{\tilde{\tau}}(Ftr,v12) = \mathbf{rSAM4}^{\tilde{\tau}}(Ftr,v12) + \mathrm{XtraVeg}^{\tilde{\tau}} \times \mathbf{rSAM4}^{\tilde{\tau}}(Ftr,XfFafh)$

114v.b $\times\, \boldsymbol{\Omega}_{XfFafh,v12} \times \{\boldsymbol{\Omega}_{1,Ftr} \times \mathbf{rSAM4}^{\tilde{\tau}}(Ftr,XfFafh) \times \boldsymbol{\Omega}_{XfFafh,1}\}^{-1}$

115v.a $\mathbf{rSAM4}^{\tilde{\tau}}(Ftr,XfFafh) = \mathbf{rSAM4}^{\tilde{\tau}}(Ftr,XfFafh)$

115v.b $-\, \mathbf{rSAM4}^{\tilde{\tau}}(Ftr,XfFafh) \times \{[\boldsymbol{\Omega}_{1,Ftr} \times \mathbf{rSAM4}^{\tilde{\tau}}(Ftr,XfFafh)]''\}^{-1}$

115v.c $\times\, (\mathrm{XtraVeg}^{\tilde{\tau}} \times [\boldsymbol{\Omega}_{1,veg} \times \mathbf{FSV}^{\tilde{\tau}}(veg,XfFafh)])''$

116v. $\mathbf{rSAM4}^{\tilde{\tau}}(v12,Xf1114) = \boldsymbol{\Omega}_{v12,FC} \times \mathbf{rSAM4}^{\tilde{\tau}}(FC,v12) \times \boldsymbol{\Omega}_{v12,Xf1114}$

Except for the added steps to account for multiple vegetable commodities, equations 107v–116v for vegetables were described the same way as equations 107f–116f were described for fruits.

Aquaculture Procurement for Packaging and for Final Market Sales

Another important protein source that is included in the other animal products commodity includes farm raised fish and seafood. We identified outlays on other animal products (*c010*) by the seafood product preparation and packaging activity (*a049* and by PCE expenditures on fish and seafood (*Xf1107*)). Additionally, a portion of foodservice establishment outlays on other animal products includes farm raised fish procurement and we estimated this value to equal a proportion of the value for outlays assigned to column *Xf1107* based on the ratio of annual fish consumption at restaurants verses at home according to current data from the National Health and Nutrition Examination Survey (Centers for Disease Control and Prevention, National Health and Nutrition Examination Survey, 2020). Lastly, a portion of exports for other animal products may represent farm raised fish. However, USDA, National Agricultural Statistics Service's (NASS) Census of Aquaculture data between 2005 and 2018 indicated no exports (USDA, National Agricultural Statistics Service (NASS), 2019).

Figure 5 is the updated schematic for Ag-FEDS **rSAM** account in its final form with the partitioned foodservice caloric commodity aggregation activities and commodities and the added voucher subaccounts (for a complete list, see table 4). Beyond the depiction of these added sections and characterizations of all subaccount transactions, figure 5 shows a new partition of the endogenous transactions submatrix (**Z**). Specifically, the **Z** submatrix is partitioned into four superquadrants that are as follow: (1) $\mathbf{Z}_{in,in}$, (2) $\mathbf{Z}_{in,out}$, (3) $\mathbf{Z}_{out,in}$, and (4) $\mathbf{Z}_{out,out}$. These partitions are important for how we compiled our multiplier models and more can be said about these partitions when applications are developed. Any aggregations, reallocations, and extensions of this account are for application specific model datasets (figure 5).

Chapter 4: Conclusion

This technical bulletin introduced the Agri-Food Economic Data System (Ag-FEDS). Ag-FEDS was developed to improve the clarity and accuracy of agricultural and food economy data when measuring how all production is distributed among consumers, businesses, governments, and global nations. To facilitate replication, we described primary data sources and every calculation, written entirely in matrix algebra.

This report first introduced the social accounting matrix (SAM) methodology and we have provided extensive documentation to show the development of Ag-FEDS. Ag-FEDS extends and refines previous work in several significant ways as described throughout Chapter 3. The result of Ag-FEDS is an expanded scope and sharpened focus of the data used to model the structure and organization of the U.S. food economy.

There are five innovations presented towards attaining this goal. First, we developed an optimal aggregation of U.S. Department of Commerce, Bureau of Economic Analysis (BEA) and Bureau of the Census (Census Bureau) source data to our *F* accounts, or target Ag-FEDS dataset using a constrained maximum likelihood mathematical programming model. Secondly, we used redefinitions and vouchers to address the issues of commodity flows that were locked into the fixed production technologies. Thirdly, we reconciled discrepancies in USDA, ERS' and BEA's food spending measures using the voucher approach. Lastly, we applied finetuning techniques for use in food system analysis. These efforts improved the foundational Ag-FEDS data to best characterize the agrifood system in the United States. Ag-FEDS is a modeling platform that can be the basis for future modeling efforts and provide a detailed accounting of food and beverage costs or resource use along the sequence of activities from farm production through points of purchase.

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Appendix

This section reports descriptive data about the Ag-FEDS data system.

Table A.1
Crosswalk of tier 0 account elements

D	Us	Su	Se	FA	FC	F account description
1111a0	111	111CA	11	a001	c001	Oilseed farming
1111B0	111	111CA	11	a002	c002	Grain farming
111200	111	111CA	11	a003	c003	Vegetable and melon farming
111300	111	111CA	11	a004	c004	Fruit and tree nut farming
111400	111	111CA	11	a005	c005	Greenhouse, nursery, and floriculture production
111900	111	111CA	11	a006	c006	Other crop farming
112120	112	111CA	11	a007	c007	Dairy cattle and milk production
1121a0	112	111CA	11	a008	c008	Beef cattle ranching and farming, including feedlots and dual-purpose ranching and farming
112300	112	111CA	11	a009	c009	Poultry and egg production
112a00	112	111CA	11	a010	c010	Animal production, except cattle and poultry and eggs
113000	113FF	113FF	11	a011	c011	Forestry and logging
114000	113FF	113FF	11	a012	c012	Fishing, hunting and trapping
115000	113FF	113FF	11	a013	c013	Support activities for agriculture and forestry
211000	211	211	21	a014	c014	Oil and gas extraction
212100	212	212	21	a015	c015	Coal mining
212230	212	212	21	a016	c016	Copper, nickel, lead, and zinc mining
2122a0	212	212	21	a018	c018	Iron, gold, silver, and other metal ore mining
212310	212	212	21	a017	c017	Stone mining and quarrying
2123a0	212	212	21	a019	c019	Other nonmetallic mineral mining and quarrying
213111	213	213	21	a020	c020	Drilling oil and gas wells
21311A	213	213	21	a021	c021	Other support activities for mining
221100	2211	22	22	a022	c022	Electric power generation, transmission, and distribution
221200	2212NW	22	22	a023	c023	Natural gas distribution
221300	2212NW	22	22	a024	c024	Water, sewage and other systems
230301	23MR	23	23	a025	c025	Maintenance and repair construction
230302	23MR	23	23	a025	c025	Maintenance and repair construction
233210	23EH	23	23	a026	c026	Education, hospital, and health structures
233230	23OT	23	23	a027	c027	Other nonresidential structures
233240	23PC	23	23	a028	c028	Power and communication structures
233262	23EH	23	23	a026	c026	Education, hospital, and health structures

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Table A.1 (cont.)

Crosswalk of tier 0 account elements

D	Us	Su	Se	FA	FC	F account description
2332a0	23OC	23	23	a029	c029	Office and commercial structures
2332c0	23TH	23	23	a030	c030	Transportation structures and highways and streets
2332D0	23OT	23	23	a027	c027	Other nonresidential structures
233411	23SF	23	23	a031	c031	Single-family residential structures
233412	23OR	23	23	a032	c032	Other residential construction
2334a0	23OR	23	23	a032	c032	Other residential construction
311111	311	311FT	31ND	a033	c033	Dog and cat food manufacturing
311119	311	311FT	31ND	a034	c034	Other animal food manufacturing
311210	311	311FT	31ND	a035	c035	Flour milling and malt manufacturing
311221	311	311FT	31ND	a036	c036	Wet corn milling
311224	311	311FT	31ND	a037	c037	Soybean and other oilseed processing
311225	311	311FT	31ND	a038	c038	Fats and oils refining and blending
311230	311	311FT	31ND	a039	c039	Breakfast cereal manufacturing
311300	311	311FT	31ND	a040	c040	Sugar and confectionery product manufacturing
311410	311	311FT	31ND	a041	c041	Frozen food manufacturing
311420	311	311FT	31ND	a042	c042	Fruit and vegetable canning, pickling, and drying
311513	311	311FT	31ND	a043	c043	Cheese manufacturing
311514	311	311FT	31ND	a044	c044	Dry, condensed, and evaporated dairy product manufacturing
31151A	311	311FT	31ND	a045	c045	Fluid milk and butter manufacturing
311520	311	311FT	31ND	a046	c046	Ice cream and frozen dessert manufacturing
311615	311	311FT	31ND	a047	c047	Poultry processing
31161A	311	311FT	31ND	a048	c048	Animal (except poultry) slaughtering, rendering, and processing
311700	311	311FT	31ND	a049	c049	Seafood product preparation and packaging
311810	311	311FT	31ND	a050	c050	Bread and bakery product manufacturing
3118a0	311	311FT	31ND	a051	c051	Cookie, cracker, pasta, and tortilla manufacturing
311910	311	311FT	31ND	a052	c052	Snack food manufacturing
311920	311	311FT	31ND	a053	c053	Coffee and tea manufacturing
311930	311	311FT	31ND	a054	c054	Flavoring syrup and concentrate manufacturing
311940	311	311FT	31ND	a055	c055	Seasoning and dressing manufacturing
311990	311	311FT	31ND	a056	c056	All other food manufacturing
312110	3121	311FT	31ND	a057	c057	Soft drink and ice manufacturing
312120	3121	311FT	31ND	a058	c058	Breweries
312130	3121	311FT	31ND	a059	c059	Wineries
312140	3121	311FT	31ND	a060	c060	Distilleries

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Table A.1 (cont.)

Crosswalk of tier 0 account elements

D	Us	Su	Se	FA	FC	F account description
312200	3122	311FT	31ND	a061	c061	Tobacco product manufacturing
313100	313TT	313TT	31ND	a062	c062	Fiber, yarn, and thread mills
313200	313TT	313TT	31ND	a063	c063	Fabric mills
313300	313TT	313TT	31ND	a064	c064	Textile and fabric finishing and fabric coating mills
314110	313TT	313TT	31ND	a065	c065	Carpet and rug mills
314120	313TT	313TT	31ND	a066	c066	Curtain and linen mills
314900	313TT	313TT	31ND	a067	c067	Other textile product mills
315000	315AL	315AL	31ND	a068	c068	Apparel manufacturing
316000	315AL	315AL	31ND	a069	c069	Leather and allied product manufacturing
321100	321	321	33DG	a070	c070	Sawmills and wood preservation
321200	321	321	33DG	a071	c071	Veneer, plywood, and engineered wood product manufacturing
321910	321	321	33DG	a072	c072	Millwork
3219a0	321	321	33DG	a073	c073	All other wood product manufacturing
322110	322	322	31ND	a074	c074	Pulp mills
322120	322	322	31ND	a075	c075	Paper mills
322130	322	322	31ND	a076	c076	Paperboard mills
322210	322	322	31ND	a077	c077	Paperboard container manufacturing
322220	322	322	31ND	a078	c078	Paper Bag and Coated and Treated Paper Manufacturing
322230	322	322	31ND	a079	c079	Stationery product manufacturing
322291	322	322	31ND	a080	c080	Sanitary paper product manufacturing
322299	322	322	31ND	a081	c081	All other converted paper product manufacturing
323110	323	323	31ND	a082	c082	Printing
323120	323	323	31ND	a083	c083	Support activities for printing
324110	324	324	31ND	a084	c084	Petroleum refineries
324121	324	324	31ND	a085	c085	Asphalt paving mixture and block manufacturing
324122	324	324	31ND	a086	c086	Asphalt shingle and coating materials manufacturing
324190	324	324	31ND	a087	c087	Other petroleum and coal products manufacturing
325110	3251	325	31ND	a088	c088	Petrochemical manufacturing
325120	3251	325	31ND	a089	c089	Industrial gas manufacturing
325130	3251	325	31ND	a090	c090	Synthetic dye and pigment manufacturing
325180	3251	325	31ND	a091	c091	Other Basic Inorganic Chemical Manufacturing
325190	3251	325	31ND	a092	c092	Other basic organic chemical manufacturing

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Table A.1 (cont.)

Crosswalk of tier 0 account elements

D	Us	Su	Se	FA	FC	F account description
325211	3252	325	31ND	a093	c093	Plastics material and resin manufacturing
3252a0	3252	325	31ND	a094	c094	Synthetic rubber and artificial and synthetic fibers and filaments manufacturing
325310	325X	325	31ND	a095	c095	Fertilizer manufacturing
325320	325X	325	31ND	a096	c096	Pesticide and other agricultural chemical manufacturing
325411	3254	325	31ND	a097	c097	Medicinal and botanical manufacturing
325412	3254	325	31ND	a098	c098	Pharmaceutical preparation manufacturing
325413	3254	325	31ND	a099	c099	In-vitro diagnostic substance manufacturing
325414	3254	325	31ND	a100	c100	Biological product (except diagnostic) manufacturing
325510	325X	325	31ND	a101	c101	Paint and coating manufacturing
325520	325X	325	31ND	a102	c102	Adhesive manufacturing
325610	325X	325	31ND	a103	c103	Soap and cleaning compound manufacturing
325620	325X	325	31ND	a104	c104	Toilet preparation manufacturing
325910	325X	325	31ND	a105	c105	Printing ink manufacturing
3259a0	325X	325	31ND	a106	c106	All other chemical product and preparation manufacturing
326110	326	326	31ND	a107	c107	Plastics packaging materials and unlaminated film and sheet manufacturing
326120	326	326	31ND	a108	c108	Plastics pipe, pipe fitting, and unlaminated profile shape manufacturing
326130	326	326	31ND	a109	c109	Laminated plastics plate, sheet (except packaging), and shape manufacturing
326140	326	326	31ND	a110	c110	Polystyrene foam product manufacturing
326150	326	326	31ND	a111	c111	Urethane and other foam product (except polystyrene) manufacturing
326160	326	326	31ND	a112	c112	Plastics bottle manufacturing
326190	326	326	31ND	a113	c113	Other plastics product manufacturing
326210	326	326	31ND	a114	c114	Tire manufacturing
326220	326	326	31ND	a115	c115	Rubber and plastics hoses and belting manufacturing
326290	326	326	31ND	a116	c116	Other rubber product manufacturing
327100	327	327	33DG	a117	c117	Clay product and refractory manufacturing
327200	327	327	33DG	a118	c118	Glass and glass product manufacturing
327310	327	327	33DG	a119	c119	Cement manufacturing
327320	327	327	33DG	a120	c120	Ready-mix concrete manufacturing
327330	327	327	33DG	a121	c121	Concrete pipe, brick, and block manufacturing
327390	327	327	33DG	a122	c122	Other concrete product manufacturing
327400	327	327	33DG	a123	c123	Lime and gypsum product manufacturing

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Table A.1 (cont.)

Crosswalk of tier 0 account elements

D	Us	Su	Se	FA	FC	F account description
327910	327	327	33DG	a124	c124	Abrasive product manufacturing
327991	327	327	33DG	a125	c125	Cut stone and stone product manufacturing
327992	327	327	33DG	a126	c126	Ground or treated mineral and earth manufacturing
327993	327	327	33DG	a127	c127	Mineral wool manufacturing
327999	327	327	33DG	a128	c128	Miscellaneous nonmetallic mineral products
331110	3311IS	331	33DG	a129	c129	Iron and steel mills and ferroalloy manufacturing
331200	3311IS	331	33DG	a130	c130	Steel product manufacturing from purchased steel
331313	3313NF	331	33DG	a131	c131	Alumina refining and primary aluminum production
331314	3313NF	331	33DG	a132	c132	Secondary smelting and alloying of aluminum
33131B	3313NF	331	33DG	a133	c133	Aluminum product manufacturing from purchased aluminum
331410	3313NF	331	33DG	a134	c134	Nonferrous Metal (except Aluminum) Smelting and Refining
331420	3313NF	331	33DG	a135	c135	Copper rolling, drawing, extruding and alloying
331490	3313NF	331	33DG	a136	c136	Nonferrous metal (except copper and aluminum) rolling, drawing, extruding and alloying
331510	3313NF	331	33DG	a137	c137	Ferrous metal foundries
331520	3313NF	331	33DG	a138	c138	Nonferrous metal foundries
332114	332	332	33DG	a139	c139	Custom roll forming
332119	332	332	33DG	a140	c140	Metal crown, closure, and other metal stamping (except automotive)
33211A	332	332	33DG	a141	c141	All other forging, stamping, and sintering
332200	332	332	33DG	a142	c142	Cutlery and handtool manufacturing
332310	332	332	33DG	a143	c143	Plate work and fabricated structural product manufacturing
332320	332	332	33DG	a144	c144	Ornamental and architectural metal products manufacturing
332410	332	332	33DG	a145	c145	Power boiler and heat exchanger manufacturing
332420	332	332	33DG	a146	c146	Metal tank (heavy gauge) manufacturing
332430	332	332	33DG	a147	c147	Metal can, box, and other metal container (light gauge) manufacturing
332500	332	332	33DG	a148	c148	Hardware manufacturing
332600	332	332	33DG	a149	c149	Spring and wire product manufacturing
332710	332	332	33DG	a150	c150	Machine shops

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Table A.1 (cont.)

Crosswalk of tier 0 account elements

D	Us	Su	Se	FA	FC	F account description
332720	332	332	33DG	a151	c151	Turned product and screw, nut, and bolt manufacturing
332800	332	332	33DG	a152	c152	Coating, engraving, heat treating and allied activities
332913	332	332	33DG	a153	c153	Plumbing fixture fitting and trim manufacturing
33291A	332	332	33DG	a154	c154	Valve and fittings other than plumbing
332991	332	332	33DG	a155	c155	Ball and roller bearing manufacturing
332996	332	332	33DG	a156	c156	Fabricated pipe and pipe fitting manufacturing
332999	332	332	33DG	a157	c157	Other fabricated metal manufacturing
33299A	332	332	33DG	a158	c158	Ammunition, arms, ordnance, and accessories manufacturing
333111	33311	333	33DG	a159	c159	Farm machinery and equipment manufacturing
333112	33311	333	33DG	a160	c160	Lawn and garden equipment manufacturing
333120	33312	333	33DG	a161	c161	Construction machinery manufacturing
333130	33313	333	33DG	a162	c162	Mining and oil and gas field machinery manufacturing
333242	3332OM	333	33DG	a163	c163	Semiconductor machinery manufacturing
33329A	3332OM	333	33DG	a164	c164	Other industrial machinery manufacturing
333314	3332OM	333	33DG	a165	c165	Optical instrument and lens manufacturing
333316	3332OM	333	33DG	a166	c166	Photographic and photocopying equipment manufacturing
333318	3332OM	333	33DG	a167	c167	Other commercial and service industry machinery manufacturing
333413	3332OM	333	33DG	a168	c168	Industrial and commercial fan and blower and air purification equipment manufacturing
333414	3332OM	333	33DG	a169	c169	Heating equipment (except warm air furnaces) manufacturing
333415	3332OM	333	33DG	a170	c170	Air conditioning, refrigeration, and warm air heating equipment manufacturing
333511	3332OM	333	33DG	a171	c171	Industrial mold manufacturing
333514	3332OM	333	33DG	a172	c172	Special tool, die, jig, and fixture manufacturing
333517	3332OM	333	33DG	a173	c173	Machine tool manufacturing
33351B	3332OM	333	33DG	a174	c174	Cutting and machine tool accessory, rolling mill, and other metalworking machinery manufacturing
333611	3332OM	333	33DG	a175	c175	Turbine and turbine generator set units manufacturing
333612	3332OM	333	33DG	a176	c176	Speed changer, industrial high-speed drive, and gear manufacturing

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Table A.1 (cont.)

Crosswalk of tier 0 account elements

D	Us	Su	Se	FA	FC	F account description
333613	3332OM	333	33DG	a177	c177	Mechanical power transmission equipment manufacturing
333618	3332OM	333	33DG	a178	c178	Other engine equipment manufacturing
333912	3332OM	333	33DG	a179	c179	Air and gas compressor manufacturing
33391A	3332OM	333	33DG	a180	c180	Pump and pumping equipment manufacturing
333920	3332OM	333	33DG	a181	c181	Material handling equipment manufacturing
333991	3332OM	333	33DG	a182	c182	Power-driven handtool manufacturing
333993	3332OM	333	33DG	a183	c183	Packaging machinery manufacturing
333994	3332OM	333	33DG	a184	c184	Industrial process furnace and oven manufacturing
33399A	3332OM	333	33DG	a185	c185	Other general purpose machinery manufacturing
33399B	3332OM	333	33DG	a186	c186	Fluid power process machinery
334111	3341	334	33DG	a187	c187	Electronic computer manufacturing
334112	3341	334	33DG	a188	c188	Computer storage device manufacturing
334118	3341	334	33DG	a189	c189	Computer terminals and other computer peripheral equipment manufacturing
334210	3342	334	33DG	a190	c190	Telephone apparatus manufacturing
334220	3342	334	33DG	a191	c191	Broadcast and wireless communications equipment
334290	3342	334	33DG	a192	c192	Other communications equipment manufacturing
334300	334X	334	33DG	a193	c193	Audio and video equipment manufacturing
334413	3344	334	33DG	a194	c194	Semiconductor and related device manufacturing
334418	3344	334	33DG	a195	c195	Printed circuit assembly (electronic assembly) manufacturing
33441A	3344	334	33DG	a196	c196	Other electronic component manufacturing
334510	3345	334	33DG	a197	c197	Electromedical and electrotherapeutic apparatus manufacturing
334511	3345	334	33DG	a198	c198	Search, detection, and navigation instruments manufacturing
334512	3345	334	33DG	a199	c199	Automatic environmental control manufacturing
334513	3345	334	33DG	a200	c200	Industrial process variable instruments manufacturing
334514	3345	334	33DG	a201	c201	Totalizing fluid meter and counting device manufacturing
334515	3345	334	33DG	a202	c202	Electricity and signal testing instruments manufacturing
334516	3345	334	33DG	a203	c203	Analytical laboratory instrument manufacturing

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Table A.1 (cont.)

Crosswalk of tier 0 account elements

D	Us	Su	Se	FA	FC	F account description
334517	3345	334	33DG	a204	c204	Irradiation apparatus manufacturing
33451A	3345	334	33DG	a205	c205	Watch, clock, and other measuring and controlling device manufacturing
334610	334X	334	33DG	a206	c206	Manufacturing and reproducing magnetic and optical media
335110	335	335	33DG	a207	c207	Electric lamp bulb and part manufacturing
335120	335	335	33DG	a208	c208	Lighting fixture manufacturing
335210	335	335	33DG	a209	c209	Small electrical appliance manufacturing
335221	335	335	33DG	a210	c210	Household cooking appliance manufacturing
335222	335	335	33DG	a211	c211	Household refrigerator and home freezer manufacturing
335224	335	335	33DG	a212	c212	Household laundry equipment manufacturing
335228	335	335	33DG	a213	c213	Other major household appliance manufacturing
335311	335	335	33DG	a214	c214	Power, distribution, and specialty transformer manufacturing
335312	335	335	33DG	a215	c215	Motor and generator manufacturing
335313	335	335	33DG	a216	c216	Switchgear and switchboard apparatus manufacturing
335314	335	335	33DG	a217	c217	Relay and industrial control manufacturing
335911	335	335	33DG	a218	c218	Storage battery manufacturing
335912	335	335	33DG	a219	c219	Primary battery manufacturing
335920	335	335	33DG	a220	c220	Communication and energy wire and cable manufacturing
335930	335	335	33DG	a221	c221	Wiring device manufacturing
335991	335	335	33DG	a222	c222	Carbon and graphite product manufacturing
335999	335	335	33DG	a223	c223	All other miscellaneous electrical equipment and component manufacturing
336111	336111	3361MV	33DG	a224	c224	Automobile manufacturing
336112	336112	3361MV	33DG	a225	c225	Light truck and utility vehicle manufacturing
336120	33612	3361MV	33DG	a226	c226	Heavy duty truck manufacturing
336211	3362BP	3361MV	33DG	a227	c227	Motor vehicle body manufacturing
336212	3362BP	3361MV	33DG	a228	c228	Truck trailer manufacturing
336213	3362BP	3361MV	33DG	a229	c229	Motor home manufacturing
336214	3362BP	3361MV	33DG	a230	c230	Travel trailer and camper manufacturing
336310	3362BP	3361MV	33DG	a231	c231	Motor vehicle gasoline engine and engine parts manufacturing
336320	3362BP	3361MV	33DG	a232	c232	Motor vehicle electrical and electronic equipment manufacturing

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Table A.1 (cont.)

Crosswalk of tier 0 account elements

D	Us	Su	Se	FA	FC	F account description
336350	3362BP	3361MV	33DG	a233	c233	Motor vehicle transmission and power train parts manufacturing
336360	3362BP	3361MV	33DG	a234	c234	Motor vehicle seating and interior trim manufacturing
336370	3362BP	3361MV	33DG	a235	c235	Motor vehicle metal stamping
336390	3362BP	3361MV	33DG	a236	c236	Other Motor Vehicle Parts Manufacturing
3363a0	3362BP	3361MV	33DG	a237	c237	Motor vehicle steering, suspension component (except spring), and brake systems manufacturing
336411	3364	3364OT	33DG	a238	c238	Aircraft manufacturing
336412	3364	3364OT	33DG	a239	c239	Aircraft engine and engine parts manufacturing
336413	3364	3364OT	33DG	a240	c240	Other aircraft parts and auxiliary equipment manufacturing
336414	3364	3364OT	33DG	a241	c241	Guided missile and space vehicle manufacturing
33641A	3364	3364OT	33DG	a242	c242	Propulsion units and parts for space vehicles and guided missiles
336500	3365AO	3364OT	33DG	a243	c243	Railroad rolling stock manufacturing
336611	3365AO	3364OT	33DG	a244	c244	Ship building and repairing
336612	3365AO	3364OT	33DG	a245	c245	Boat building
336991	3365AO	3364OT	33DG	a246	c246	Motorcycle, bicycle, and parts manufacturing
336992	3365AO	3364OT	33DG	a247	c247	Military armored vehicle, tank, and tank component manufacturing
336999	3365AO	3364OT	33DG	a248	c248	All other transportation equipment manufacturing
337110	337	337	33DG	a249	c249	Wood kitchen cabinet and countertop manufacturing
337121	337	337	33DG	a250	c250	Upholstered household furniture manufacturing
337122	337	337	33DG	a251	c251	Nonupholstered wood household furniture manufacturing
337127	337	337	33DG	a252	c252	Institutional furniture manufacturing
33712N	337	337	33DG	a253	c253	Other household nonupholstered furniture
337215	337	337	33DG	a254	c254	Showcase, partition, shelving, and locker manufacturing
33721A	337	337	33DG	a255	c255	Office furniture and custom architectural woodwork and millwork manufacturing
337900	337	337	33DG	a256	c256	Other furniture related product manufacturing
339112	3391	339	33DG	a257	c257	Surgical and medical instrument manufacturing
339113	3391	339	33DG	a258	c258	Surgical appliance and supplies manufacturing

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Table A.1 (cont.)

Crosswalk of tier 0 account elements

D	Us	Su	Se	FA	FC	F account description
339114	3391	339	33DG	a259	c259	Dental equipment and supplies manufacturing
339115	3391	339	33DG	a260	c260	Ophthalmic goods manufacturing
339116	3391	339	33DG	a261	c261	Dental laboratories
339910	3399	339	33DG	a262	c262	Jewelry and silverware manufacturing
339920	3399	339	33DG	a263	c263	Sporting and athletic goods manufacturing
339930	3399	339	33DG	a264	c264	Doll, toy, and game manufacturing
339940	3399	339	33DG	a265	c265	Office supplies (except paper) manufacturing
339950	3399	339	33DG	a266	c266	Sign manufacturing
339990	3399	339	33DG	a267	c267	All other miscellaneous manufacturing
4200ID	42ID	42	42	a268	c268	Customs duties
423100	4231	42	42	a269	c269	Motor vehicle and motor vehicle parts and supplies
423400	4234	42	42	a270	c270	Professional and commercial equipment and supplies
423600	4236	42	42	a271	c271	Household appliances and electrical and electronic goods
423800	4238	42	42	a272	c272	Machinery, equipment, and supplies
423a00	423X	42	42	a273	c273	Other durable goods merchant wholesalers
424200	4242	42	42	a274	c274	Drugs and druggists' sundries
424400	4244	42	42	a275	c275	Grocery and related product wholesalers
424700	4247	42	42	a276	c276	Petroleum and petroleum products
424a00	424X	42	42	a277	c277	Other nondurable goods merchant wholesalers
425000	425	42	42	a278	c278	Wholesale electronic markets and agents and brokers
441000	441	441	44RT	a279	c279	Motor vehicle and parts dealers
444000	444	4a0	44RT	a280	c280	Building material and garden equipment and supplies dealers
445000	445	445	44RT	a281	c281	Food and beverage stores
446000	446	4a0	44RT	a282	c282	Health and personal care stores
447000	447	4a0	44RT	a283	c283	Gasoline stations
448000	448	4a0	44RT	a284	c284	Clothing and clothing accessories stores
452000	452	452	44RT	a285	c285	General merchandise stores
454000	454	4a0	44RT	a286	c286	Nonstore retailers
481000	481	481	48TW	a288	c288	Air transportation
482000	482	482	48TW	a289	c289	Rail transportation
483000	483	483	48TW	a290	c290	Water transportation
484000	484	484	48TW	a291	c291	Truck transportation
485000	485	485	48TW	a292	c292	Transit and ground passenger transportation

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Table A.1 (cont.)

Crosswalk of tier 0 account elements

D	Us	Su	Se	FA	FC	F account description
486000	486	486	48TW	a293	c293	Pipeline transportation
48a000	48A	487OS	48TW	a294	c294	Scenic and sightseeing transportation and support activities for transportation
491000	GFE	GFE	G	a351	c351	Federal government enterprises
492000	492	487OS	48TW	a295	c295	Couriers and messengers
493000	493	493	48TW	a296	c296	Warehousing and storage
4B0000	4a0X	4a0	44RT	a287	c287	All other retail
511110	5111	511	51	a297	c297	Newspaper, periodical, book, and directory publishers
511120	5111	511	51	a297	c297	Newspaper, periodical, book, and directory publishers
511130	5111	511	51	a297	c297	Newspaper, periodical, book, and directory publishers
5111a0	5111	511	51	a297	c297	Newspaper, periodical, book, and directory publishers
511200	5112	511	51	a298	c298	Software publishers
512100	512	512	51	a299	c299	Motion picture and sound recording industries
512200	512	512	51	a299	c299	Motion picture and sound recording industries
515100	515	513	51	a300	c300	Broadcasting (except Internet)
515200	515	513	51	a300	c300	Broadcasting (except Internet)
517110	5171	513	51	a301	c301	Wired telecommunications carriers
517210	5172	513	51	a302	c302	Wireless telecommunications carriers (except satellite)
517a00	5174OT	513	51	a303	c303	Other telecommunications, including satellite
518200	518	514	51	a304	c304	Data processing, hosting, and related services
519130	519	514	51	a305	c305	Other information services
5191a0	519	514	51	a305	c305	Other information services
522a00	521CI	521CI	52	a306	c306	Federal Reserve banks, credit intermediation, and related activities
523900	523	523	52	a307	c307	Securities, commodity contracts, and investments
523a00	523	523	52	a307	c307	Securities, commodity contracts, and investments
524113	524113	524	52	a308	c308	Direct life insurance carriers
5241XX	5241X	524	52	a309	c309	Insurance carriers, except direct life
524200	5242	524	52	a310	c310	Agencies, brokerages, and other insurance related activities
525000	525	525	52	a311	c311	Funds, trusts, and other financial vehicles
52a000	521CI	521CI	52	a306	c306	Federal Reserve banks, credit intermediation, and related activities

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Table A.1 (cont.)

Crosswalk of tier 0 account elements

D	Us	Su	Se	FA	FC	F account description
531HSO	HSO	HS	53	a312	c312	Owner-occupied housing
531HST	HST	HS	53	a313	c313	Tenant-occupied housing
531ORE	ORE	ORE	53	a314	c314	Other real estate
532100	532RL	532RL	53	a315	c315	Rental and leasing services and lessors of intangible assets
532400	532RL	532RL	53	a315	c315	Rental and leasing services and lessors of intangible assets
532a00	532RL	532RL	53	a315	c315	Rental and leasing services and lessors of intangible assets
533000	532RL	532RL	53	a315	c315	Rental and leasing services and lessors of intangible assets
541100	5411	5411	54	a316	c316	Legal services
541200	5412	5412OP	54	a317	c317	Accounting, tax preparation, bookkeeping, and payroll services
541300	5413	5412OP	54	a318	c318	Architectural, engineering, and related services
541400	541X	5412OP	54	a319	c319	Specialized design services and other professional, scientific, and technical services
541511	5415	5415	54	a320	c320	Computer systems design and related services
541512	5415	5415	54	a320	c320	Computer systems design and related services
54151A	5415	5415	54	a320	c320	Computer systems design and related services
541610	5416	5412OP	54	a321	c321	Management, scientific, and technical consulting services
5416a0	5416	5412OP	54	a321	c321	Management, scientific, and technical consulting services
541700	5417	5412OP	54	a322	c322	Scientific research and development services
541800	5418	5412OP	54	a323	c323	Advertising, public relations, and related services
541920	541X	5412OP	54	a319	c319	Specialized design services and other professional, scientific, and technical services
541940	541X	5412OP	54	a319	c319	Specialized design services and other professional, scientific, and technical services
5419a0	541X	5412OP	54	a319	c319	Specialized design services and other professional, scientific, and technical services
550000	55	55	55	a324	c324	Management of companies and enterprises
561100	561X	561	56	a325	c325	Other administrative and support services
561200	561X	561	56	a325	c325	Other administrative and support services

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Table A.1 (cont.)

Crosswalk of tier 0 account elements

D	Us	Su	Se	FA	FC	F account description
561300	5613	561	56	a326	c326	Employment services
561400	561X	561	56	a325	c325	Other administrative and support services
561500	561X	561	56	a325	c325	Other administrative and support services
561600	561X	561	56	a325	c325	Other administrative and support services
561700	5617	561	56	a327	c327	Services to buildings and dwellings
561900	561X	561	56	a325	c325	Other administrative and support services
562000	562	562	56	a328	c328	Waste management and remediation services
611100	61	61	61	a329	c329	Educational services
611a00	61	61	61	a329	c329	Educational services
611B00	61	61	61	a329	c329	Educational services
621100	6211	621	62	a330	c330	Offices of physicians
621200	6212	621	62	a331	c331	Offices of dentists
621300	6213	621	62	a332	c332	Offices of other health practitioners
621400	6214	621	62	a333	c333	Outpatient care centers
621500	6215OH	621	62	a334	c334	Other ambulatory health care services
621600	6215OH	621	62	a334	c334	Other ambulatory health care services
621900	6215OH	621	62	a334	c334	Other ambulatory health care services
622000	622	622	62	a335	c335	Hospitals
623a00	623	623	62	a336	c336	Nursing and residential care facilities
623B00	623	623	62	a336	c336	Nursing and residential care facilities
624100	624	624	62	a337	c337	Social assistance
624400	624	624	62	a337	c337	Social assistance
624a00	624	624	62	a337	c337	Social assistance
711100	711AS	711AS	71	a338	c338	Performing arts, spectator sports, museums, and related activities
711200	711AS	711AS	71	a338	c338	Performing arts, spectator sports, museums, and related activities
711500	711AS	711AS	71	a338	c338	Performing arts, spectator sports, museums, and related activities
711a00	711AS	711AS	71	a338	c338	Performing arts, spectator sports, museums, and related activities
712000	711AS	711AS	71	a338	c338	Performing arts, spectator sports, museums, and related activities
713100	713	713	71	a339	c339	Amusements, gambling, and recreation industries
713200	713	713	71	a339	c339	Amusements, gambling, and recreation industries
713900	713	713	71	a339	c339	Amusements, gambling, and recreation industries
721000	721	721	72	a340	c340	Accommodation

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Table A.1 (cont.)

Crosswalk of tier 0 account elements

D	Us	Su	Se	FA	FC	F account description
722110	722	722	72	a341	c341	Full-service restaurants
722211	722	722	72	a342	c342	Limited-service restaurants
722a00	722	722	72	a343	c343	All other food and drinking places
811100	811	81	81	a344	c344	Repair and maintenance
811200	811	81	81	a344	c344	Repair and maintenance
811300	811	81	81	a344	c344	Repair and maintenance
811400	811	81	81	a344	c344	Repair and maintenance
812100	812	81	81	a345	c345	Personal and laundry services
812200	812	81	81	a345	c345	Personal and laundry services
812300	812	81	81	a345	c345	Personal and laundry services
812900	812	81	81	a345	c345	Personal and laundry services
813100	813	81	81	a346	c346	Religious, grantmaking, civic, professional, and similar organizations
813a00	813	81	81	a346	c346	Religious, grantmaking, civic, professional, and similar organizations
813B00	813	81	81	a346	c346	Religious, grantmaking, civic, professional, and similar organizations
814000	814	81	81	a347	c347	Private households
GSLGE	GSLGE	GSLG	G	a348	c348	State and local government (educational services)
GSLGH	GSLGH	GSLG	G	a349	c349	State and local government (hospitals and health services)
GSLGO	GSLGO	GSLG	G	a350	c350	State and local government (other services)
S00101	GFE	GFE	G	a351	c351	Federal government enterprises
S00102	GFE	GFE	G	a351	c351	Federal government enterprises
S00201	GSLE	GSLE	G	a352	c352	State and local government enterprises
S00202	GSLE	GSLE	G	a352	c352	State and local government enterprises
S00203	GSLE	GSLE	G	a352	c352	State and local government enterprises
S00300	S003	Other	Other	a356	c356	Noncomparable imports
S00401	S004	Used	Used	a357	c357	Scrap, used and secondhand goods
S00402	S004	Used	Used	a357	c357	Scrap, used and secondhand goods
S00500	GFGD	GFGD	G	a353	c353	Federal general government (defense)
S00600	GFGN	GFGN	G	a354	c354	Federal general government (nondefense)
S00900	S009	Other	Other	a355	c355	Rest of the world adjustment

Source: USDA, Economic Research Service (ERS) and “BEA Industry and Commodity Codes and NAICS Concordance” (U.S. Department of Commerce, Bureau of Economic Analysis, 2024g).

Table A.2

Description of tier 2 and 3 sets from the *D* and *F* accounts

Tier 0 set	Tier 1 set	Tier 2 or 3 set	Description	Tier 0 set	Tier 1 set	Tier 2 or 3 set	Description
D	*C*	*mrg*	All margin commodities	*F*	*C*	*veg*	Commodities purchased as veggies
D	*C*	*xmg*	All non-margin commodities	*F*	*C*	*cala*	Caloric alcoholic beverage commodities
D	*I*	*h*	Households	*F*	*C*	*calxa*	Caloric commodities except alcohol
D	*I*	*g*	Domestic Governments	*F*	*C*	*farm*	Farm commodities
D	*I*	*k*	Capital/financial	*F*	*C*	*chem*	Chemical commodities sold as food
D	*R*	*r1*	Exports	*F*	*C*	*mg2*	Margin commodities including foodservices and voucher overhead
D	*R*	*r2*	Imports	*F*	*C*	*ag*	Agricultural commodities
D	*O*	*p*	Producer prices	*F*	*C*	*xag*	Commodities excluding agriculture
D	*O*	*mg*	Margin prices	*F*	*C*	*SC*	Supply chain commodities
D	*O*	*m*	Market prices	*F*	*C*	*NC*	Subcontracting commodities
F	*A*	*mfg*	All manufacturing activities	*F*	*I*	*h*	Households
F	*A*	*nfs*	Non-traditional foodservice activities	*F*	*I*	*g*	Domestic Governments
F	*A*	*fs*	Foodservice activities	*F*	*I*	*k*	Capital/financial
F	*A*	*fsc*	Caloric bundling for foodservices	*F*	*R*	*r1*	Exports
F	*A*	$\mu\lambda$	All activities using voucher lambda	*F*	*R*	*r2*	Imports
F	*A*	*xfs*	All activities except foodservices	*F*	*E*	*fbah*	All food and beverages at home *Xf* accounts
F	*A*	*fwfr*	Food wholesale and retail	*F*	*E*	*Xf*	All food dollar table accounts
F	*A*	*owor*	Other nondurable wholesale and retail	*F*	*E*	*Xffah*	All food at home *Xf* accounts
F	*A*	*mp*	Meat processing activity	*F*	*E*	*Xfaah*	All alcoholic beverages at home *Xf* accounts
F	*A*	*xmp*	Activities except meat processing	*F*	*E*	*XfFaw*	Food at work *Xf* account
F	*A*	*x045*	Activities except fluid milk manufacturing	*F*	*E*	*Xfsm*	School meals *Xf* account
F	*A*	*SA*	Supply chain activities	*F*	*E*	*Xflefa*	Institutional/employer furnished and food assistance *Xf* account
F	*A*	*NA*	Subcontracting activities	*F*	*E*	*XfFafh*	All food away from home *Xf* accounts
F	*C*	*mrg*	All margin commodities	*F*	*O*	*p*	Producer prices

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Table A.2 (cont.)

Description of tier 2 and 3 sets from the *D* and *F* accounts

Tier 0 set	Tier 1 set	Tier 2 or 3 set	Description	Tier 0 set	Tier 1 set	Tier 2 or 3 set	Description
F	*C*	*xmg*	All commodities except margin coms	*F*	*O*	*m*	Market prices
F	*C*	*cen*	Commodities covered by census trade	*F*	*V*	*psb*	Price spread beef for retail
F	*C*	*xcen*	Commodities not covered by census	*F*	*V*	*psp*	Price spread pork for retail
F	*C*	*nfs*	Non-traditional foodservice commodities	*F*	*V*	*psm*	Price spread fluid milk for retail
F	*C*	*fnb*	Food and beverage commodities linked to *Anfs*	*F*	*V*	*psf*	Price spread fresh fruits for retail
F	*C*	*fs*	Commercial foodservices	*F*	*V*	*psv*	Price spread fresh veggies for retail
F	*C*	*cal*	Caloric commodities linked to *Afs*	*F*	*V*	*psom*	Price spread other meats
F	*C*	*fsc*	Caloric bundling for foodservices	*F*	*V*	*psod*	Price spread other dairy
F	*C*	*fwfr*	Food wholesale and retail	*F*	*V*	*pro*	Commodity procurement vouchers
F	*C*	*owor*	Other nondurable wholesale and retail	*F*	*V*	*SV*	Supply chain vouchers
F	*C*	*fr*	Food retail	*F*	*X*	*meat*	All meat final demand accounts
F	*C*	*or*	Other nondurable retail	*F*	*X*	*xmeat*	All nonmeat final demand accounts
F	*C*	*trn*	Transportation margin commodities	*F*	*X*	*xmilk*	All nonmilk final demand accounts

Note: *D* Account = BEA Deatail Industry/Commodity. *F* Account = Ag-FEDS account.
Source: USDA, Economic Research Service (ERS).