Appendix

Decomposition of total factor productivity

This study uses a stochastic frontier analysis to decompose TFP growth into four components: (1) technical change, which is the increase in the maximum output that can be produced from a given level of inputs (a shift in the production frontier); (2) technical efficiency change, which is the change in a firm's ability to achieve maximum output given its set of inputs (how close it is to the production frontier); (3) scale efficiency change, which is the change in the degree to which a firm is optimizing the scale of its operations; and (4) allocative efficiency change, which is the change in a firm's ability to select a level of inputs so as to ensure that the input price ratios equal the ratios of the corresponding marginal products.[13]

[13]The derivations in this appendix are based primarily on Orea (2002); Coelli et al. (2005), pp. 289-302; and Coelli et al. (2003), pp. 25-66.

Orea (2002) shows that if firm *i*'s technology in time *t* can be represented by the translog output-oriented distance function $D_o(q_{it}, x_{it}, t)$ where *q* is output, x_{it}, a *K*-dimensional input vector with elements $(x_{it1}, \ldots x_{itk} \ldots x_{itK})$, then the logarithm of a generalized output-oriented Malmquist productivity index ln M_o can be decomposed into changes in technical efficiency (EC), technical change (TC), and scale efficiency change (SC), between time periods *r* and *s*:

(1) $$\ln M_{Oi} = EC_i^{rs} + TC_i^{rs} + SC_i^{rs}$$

where

(2) $$EC_i^{rs} = \ln D_O(q_{is}, x_{is}, s) - \ln D_O(q_{ir}, x_{ir}, r)$$

(3) $$TC_i^{rs} = -\frac{1}{2}\left[\frac{\partial \ln D_O(q_{is}, x_{is}, s)}{\partial t} + \frac{\partial \ln D_O(q_{ir}, x_{ir}, r)}{\partial t}\right]$$

(4) $$SC_i^{rs} = \frac{1}{2}\sum_{k=1}^{K}\left[\frac{\varepsilon_{is}-1}{\varepsilon_{is}}\varepsilon_{isk} + \frac{\varepsilon_{ir}-1}{\varepsilon_{ir}}\varepsilon_{irk}\right]\cdot \ln\left(\frac{x_{isk}}{x_{irk}}\right)$$

where $t = (r,s)$, $\varepsilon_{it} = \sum_{k=1}^{K}\varepsilon_{itk}$ is the scale elasticity such that $\varepsilon_{itk} = \partial \ln D_O(q_{it}, x_{it}, t)/\partial \ln x_{itk}$.

With one output, a translog distance function can be defined:

(5) $$\ln D_O(q_{it}, x_{it}, t) = \ln q_{it} - f(\beta, x_{it}, t) - v_{it}$$

where β is a vector of parameters, v_{it} is a normally distributed random error with mean zero and:

(6) $$f(\beta, x_{it}, t) = \beta_0 + \sum_{k=1}^{K}\beta_k \ln x_{itk} + \frac{1}{2}\sum_{k=1}^{K}\sum_{j=1}^{K}\beta_{kj}\ln x_{itk}\ln x_{itj} + \sum_{k=1}^{K}\beta_{tk} t \ln x_{itk} + \beta_t t + \frac{1}{2}\beta_{tt} t^2 .$$

To account for technical inefficiency, we estimate a stochastic production function model of the form:

(7) $$\ln q_{it} = f(\beta, x_{it}, t) + v_{it} - u_{it}$$

where u_{it}, a nonnegative random variable associated with technical inefficiency, is drawn from a truncated normal distribution (Battese and Coelli, 1992). An output-oriented measure of technical efficiency is the ratio of observed output to the corresponding stochastic frontier output:

$$(8) \qquad TE_{it} = \frac{q_{it}}{\exp\left(f(\beta, x_{it}, t) + v_{it}\right)} = \frac{\exp\left(f(\beta, x_{it}, t) + v_{it} - u_{it}\right)}{\exp\left(f(\beta, x_{it}, t) + v_{it}\right)} = \exp\left(-u_{it}\right)$$

Using (7), it can be shown that the technical efficiency factor (8) equals the distance function (5):

$$(9) \qquad \exp\left(-u_{it}\right) = \exp\left(\ln q_{it} - f(\beta, x_{it}, t) - v_{it}\right) = D_0\left(q_{it}, x_{it}, t\right).$$

The technical efficiency measure (8) can be estimated conditional on $e_{it} = v_{it} - u_{it}$. It follows from (2) and (8) that the efficiency change can be estimated:

$$(10) \qquad EC_i^{rs} = E\left(-u_{is} \mid e_{is}\right) - E\left(-u_{ir} \mid e_{ir}\right)$$

or

$$(11) \qquad \exp\left(EC_i^{rs}\right) = E\left(\exp\left(-u_{is} \mid e_{is}\right)\right) / E\left(\exp\left(-u_{ir} \mid e_{ir}\right)\right),$$

where the numerator and denominator in (11) are the estimated technical efficiency scores in periods *s* and *r*, respectively, which have values between zero and one.

Using (3), (5), and (6), the technical change index can be derived:

$$(12) \qquad TC_i^{rs} = \frac{1}{2}\left[\sum_{k=1}^{K} \beta_{tk} \ln x_{isk} + \sum_{k=1}^{K} \beta_{tk} \ln x_{irk} + 2\beta_t + 2\beta_{tt}\left(r + s\right)\right].$$

From (6), the scale elasticity is:

$$(13) \qquad \varepsilon_{it} = \sum\nolimits_{k=1}^{K} \varepsilon_{itk} \text{ , where } \varepsilon_{itk} = \beta_k + \frac{1}{2}\sum_{j=1}^{K} \beta_{kj} \ln x_{itj} + \beta_{tk} t$$

This can be used to compute the scale efficiency change index (4).

To estimate allocative efficiency change, ERS compared the Malmquist TFP index (1) to the logarithm of the Tornqvist TFP change index (with one output):

$$(14) \qquad \ln TFP_i^{rs} = \ln q_{is} - \ln q_{ir} - \frac{1}{2}\sum_{k=1}^{K}\left[\left(\sigma_{isk} + \sigma_{irk}\right) \cdot \left(\ln x_{isk} - \ln x_{irk}\right)\right]$$

where σ_{itk} are the input cost shares. Any difference between the Tornqvist TFP change calculated in (14) and the Malmquist TFP index calculated in (1) must be due to allocative efficiency change. Hence, it can be shown that the allocative efficiency change is:

$$(15) \qquad AC_i^{rs} = \frac{1}{2}\sum_{k=1}^{K}\left[\left(\left(\frac{\varepsilon_{isk}}{\varepsilon_{sk}} - \sigma_{isk}\right) + \left(\frac{\varepsilon_{irk}}{\varepsilon_{rk}} - \sigma_{irk}\right)\right) \cdot \left(\ln x_{isk} - \ln x_{irk}\right)\right]$$

In the analysis, output is defined as “hog weight gain”—the weight added to purchased/placed hogs and existing hog inventory in the calendar year prior to the year of the survey. Feed is defined as the total weight of feed applied. The labor input is a Tornqvist quantity index comprised of paid labor and unpaid farm household labor using the labor expenditure shares for paid and unpaid labor as weights. Capital is the “capital recovery cost”—the estimated cost of replacing the existing capital equipment (barns, feeding equipment, etc.). “Other inputs” is defined as expenditures on veterinary services, bedding, marketing, custom work, energy, and repairs. Labor wages are deflated using the Bureau of Labor Statistics (BLS) Blue Collar Total Compensation index; feed prices are deflated using a weighted average of the BLS corn and soybean Producer Price Index (PPI); Capital is deflated using the BLS farm machinery PPI, and other inputs are deflated using the CPI. In the estimation, ERS rescaled all logged values of the variables as deviations from the sample mean to facilitate interpretation of the coefficients.

The appendix table presents the estimated coefficients of the stochastic production function (6). Because the variables are expressed as deviations from their means, the first-order parameters of the translog function can be directly interpreted as estimates of production elasticities evaluated at the sample means. The production elasticities with respect to feed, capital, and other inputs have plausible values and are statistically significant.

Appendix table 1

Stochastic production function parameter estimates

Parameter		Coefficient	Standard error	t-statistic
β_0	constant	0.377	0.0385	9.8
β_1	feed	0.473	0.0214	22.2
β_2	labor	0.045	0.0119	3.8
β_3	capital	0.319	0.0258	12.4
β_4	other inputs	0.280	0.0193	14.5
β_{11}		0.101	0.0323	3.1
β_{22}		-0.028	0.0148	-1.9
β_{33}		0.092	0.0609	1.5
β_{44}		0.081	0.0337	2.4
β_{12}		-0.0055	0.0188	-0.3
β_{13}		-0.0791	0.0383	-2.1
β_{14}		-0.0738	0.0268	-2.8
β_{23}		0.0060	0.0207	0.3
β_{24}		-0.0183	0.0174	-1.1
β_{34}		0.0226	0.0366	0.6
β_t	time	0.0619	0.0034	18.2
β_{tt}	time-squared	0.0046	0.0017	2.7
β_{t1}		-0.0257	0.0045	-5.7
β_{t2}		0.0012	0.0029	0.4
β_{t3}		0.0065	0.0058	1.1
β_{t4}		0.0212	0.0043	4.9
σ^2	$(=\sigma^2_v + \sigma^2_u)$	0.355	0.0300	11.8
γ	$(=\sigma^2_v + \sigma^2_u)$	0.725	0.0536	13.5
Observations		1,181		

Source: USDA, ERS using data from the 1992 Farm Costs and Returns Survey and USDA's 1998 and 2004 Agricultural Resource Management Surveys.